Simultaneous and Sequential Voting under General Decision Rules

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Abstract. In an economic theory of voting, voters have positive or negative costs of voting in favor of a proposal and positive or negative benefits from an accepted proposal. When votes have equal weight then simultaneous voting mostly has a unique pure strategy Nash equilibrium which is independent of benefits. Voting with respect to (arbitrarily small) costs alone, however, often results in voting against the “true majority”. If voting is sequential as in the roll call votes of the US Senate then, in the unique subgame perfect equilibrium, the ”true majority” prevails (Groseclose and Milyo, 2010, 2013). In this paper, it is shown that the result for sequential voting holds also with different weights of voters (shareholders) or with multiple necessary majorities (EU decision making). Simultaneous voting in the general model can be plagued by non-existent or non-unique pure strategy equilibria under most preference constellations.

Keywords: Voting, free riding, binary decisions, unique equilibria

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I. Introduction

The economic and political relations of countries are more and more governed by membership in multinational organizations instead of bilateral contracts. Decision making in these organizations has to take into account considerable asymmetry concerning the size of the countries as well as their economic, political, and military power. This leads to weighted voting and the requirement of multiple majorities. Posner and Sykes (2014) provide a survey of the plethora of voting rules in international organizations and they highlight unsatisfactory implications of these rules. In this paper, I suggest an economic model of voting which generalizes results of Groseclose and Milyo (2010, 2013) for majority voting with equal weights. The model covers weighted voting by shareholders of firms, requirements of multiple majorities as in EU decision making, and all kinds of other social decision rules based on votes. The central question is whether or not there is a unique pure strategy equilibrium of the voting game and, if such an equilibrium exists, whether or not it has the same result as sincere voting[1]. Let me first explain the problem for parliamentary voting.

Imagine the members of a parliament simultaneously voting on a tax increase bill. The opposition has decided to reject the bill. All members of the ruling party or coalition as well as all members of the opposition have to fear negative consequences (bear costs) if they do not follow their party lines. Those members of the ruling party who are convinced that a tax increase would be beneficial have a dominant strategy of voting Yes. Now imagine that, in spite of the votes of these unconditional supporters, still k votes are missing for the proposal to be passed. n^{-} members of the ruling party believe that taxes should *not* be increased. They want the proposal to be rejected but they would not like to vote against it although their costs of voting No are smaller than their benefits from preventing the tax. (Otherwise also for such members of the ruling party the dominant strategy would support the proposal.) n^{+}members of the opposition are in a comparable situation. They want that the proposal passes but they would not like to support it by a positive vote.[2] If we assume the voters with dominant strategies to follow their dominant strategy then

[1] Sincere voting is the optimal vote under the assumption that a voter is decisive.
[2] The conflicts of voters without dominant strategies are described as "preference falsification" by Kumar (1995).

$n = n^{+} + n^{-}$ players without dominant strategies remain. If at least k of them support the proposal then it is accepted. If, in a simultaneous vote, $n^{+} > 0$ and $n^{-} > 0$ then we call this game among the remaining n players a narrow vote. In such games mostly a unique pure strategy equilibrium exists; otherwise there is no pure strategy equilibrium (Groseclose and Milyo, 2010).

If the unique equilibrium of a narrow vote exists, it is a "free rider equilibrium" where no player incurs costs of voting against the party line. Party whips are successful even if their threats imply only small costs for deviating party members. This success is contrary to the intention of representative democracy where a member of parliament should be guided only by his personal conscience and creed[3] expressed by the positive or negative benefits of the acceptance of a proposal. Of course there are arguments in favor of party discipline, in particular stability of the government. A regular suppression of deviating opinions seems, however, rather alarming. (See Kilgour et al., 2006.) The ruling party, however, will win votes in spite of a possibly large number of own members with contrary opinions. Without voting costs this would happen only if $n^{+} \geq k$, i.e. in cases of "true" majorities. Exactly this result is implied by sequential voting (Groseclose and Milyo, 2013).

In this paper, I want to propose a more general model of voting (including different weights and multiple majorities) and I want to extend the discussion about the origin of voting costs and about model assumptions. The variability of results under general simultaneous voting is demonstrated with three examples: a chairman with tie-breaking power, voting in the UN Security council with its veto players, and decision making in the EEC, the founding community of the EU.

Voting costs may be intrinsic (party loyalty, religious requirements, conscience) or extrinsic, for example caused by the threats of party whips. The latter requires observability of the individual votes. While some votes (for example, electing the German Bundeskanzler) are principally secret, in a survey on voting methods in Western legislatures, Saalfeld (1985) finds that perfectly secret voting is exceptional.

[3] This is formally required by the German constitutional law, but also other Western democracies formally or informally require their members of parliament to be free in their decisions.

In addition to recorded votes[4] there are many types of "semipublic" votes, for example by raising hands. If the number of possible deviators is small, however, party whips[5] can easily check their voting behavior in such circumstances.

In addition to party whips, who or what extrinsically determines the costs c_i of voting? There is first the electorate of a delegate. If she wants to be re-elected she has to take their opinion into account. This is even more important if she has *promised* to support and foster a certain politic. Promises make the delegate her own party whip. A delegate will also take into account the general public opinion and, perhaps, issues of political correctness, and the echo of her decisions in press, television and other public media.

In local parliaments (councils) private relations are stronger and public and private benefits are often more entangled than in national parliaments. The equivalents of party whips are personal friends within and outside the council. The same is true for condominium and firm owners' meetings, where there is often a strong divergence of interests. In the latter cases, however, we mostly have different weights of voters.

A country's signing or not signing a climate contracts or voting in the UN Security Council (with or without veto power) is connected with positive or negative costs in the form of public support or frustration at home and gaining or losing international reputation. Powerful countries can assume the role of party whips and urge other countries, often otherwise their allies, to follow their vote. An insinuation of reduced military protection or deterioration of bilateral economic relations create costs of voting against "big brother".

Intrinsic values of voting for or against a proposal or candidate are assumed in the vast literature on "expressive preferences" (Brennan and Lomasky, 1997; Hillman, 2010), where voters want others to show their preferences by voting in favor of a

[4] In most Western countries votes will be recorded when required. In the German parliament only personal elections and votes of no confidence are principally secret. The requirement by 5% of the members (or of a faction) of parliament are sufficient for recording the votes. The records are published in the internet.

[5] The parties' carrots and sticks are (Kilgour et al., 2006): "A 'loyal' MP who votes the party line will be a candidate for promotion (if in the government party, perhaps to Cabinet), or other benefits from the party, such as interesting trips or appointment to an interesting House committee. A 'disloyal' MP who votes against the party leadership may be prevented from ascending the political ladder and could ultimately be thrown out of the party caucus."

party and do not follow strategic considerations; but in that literature the "paradox of voting" is of central interest and not the equilibrium of voting on a certain proposal.

The literature on simultaneous and sequential voting is vast and differentiated, but there do not seem to be attempts to investigate a model with such general decision rules as outlined above. One strand of literature investigates voting without costs but with incomplete information of the voters about benefits (Dekel and Piccione, 2000). A central question in this literature is whether certain voting procedures successfully aggregate private information. The central topic in a second strand of models is strategic voting and the dynamics of agenda selection by a sequence of "formateurs" under consideration of political institutions and structures. In a classic paper by Baron and Ferejohn (1989) an agenda is the distribution of a dollar among the voters. In these models, *costless* voting on a single agenda would result in an essentially unique equilibrium. A third strand of literature is about bribing and lobbying for majorities. Closest to this investigation, however, are two papers by Gloseclose and Milyo (2010, 2013) where the results for equal weights voting are proved in a similar model. My model is simpler and more general, however. I will compare typical approaches from the bribing literature and Gloseclose and Milyo (2010, 2013) with my approach after presenting my model. In the conclusion I will mention additional literature.

In the next section I set up a general model of voting and derive general propositions and applications to special cases as weighted voting with equal and non-equal weights. In the third section (conclusion) I comment on the importance and applicability of my results.

II. Voting games

Definition 1: In a *voting game*, there are $n \geq 2$ players who simultaneously or sequentially vote "Yes" or "No" on a certain proposal. $N = \{1, \ldots, \mathrm{n}\}$ is the player set. $\mathcal{H}$ designates the set of all subsets of N whose Yes votes suffice to accept the proposal. It has the following properties: The empty set $\phi \notin \mathcal{H}$; $N \in \mathcal{H}$; if $S \subset S' \subset N$ and $S \in \mathcal{H}$ then also $S' \in \mathcal{H}$. We call $S \in \mathcal{H}$ *a minimal supporting set* if no strict subset of S is contained in $\mathcal{H}$. Player i is called a *pivot player* with respect to S if $S \cup$

$\{i\}$ is a minimal supporting set. A player i is called *negligible*[6] if she is not contained in any minimal supporting set. If $\mathcal{H} = \{S: |S| \geq k\}$, we call the game an *equal weight voting game*.

With other applications in mind we call this model a Binary Threshold Public Good game (Bolle, 2015). Definition 1 is equal to the definition of *Simple Cooperative Games* where the characteristic function takes binary values. We will establish, however, a *non-cooperative* game with more structure.

Assumption 1: If the proposal is not accepted and if a player i votes No then her revenue is $R_i = 0$, i.e. the status quo is evaluated by 0. Player i bears costs c_i if she votes “Yes" and she enjoys benefits G_i if the proposal is accepted. Players want to maximize their revenues R_i *which are benefits minus costs*, formally $R_i = G_i * 1_{Suc}$ - $c_i * 1_{Yes}$ with $1_{Suc} = 1$ if voters from a set $S \in \mathcal{H}$ vote Yes and $1_{Suc} = 0$ otherwise; $1_{Yes} = 1$ if player i has voted Yes and $1_{Yes} = 0$ otherwise.

Comparing this assumption with the introductory example, players with positive costs are from the opposition and those with negative costs are from the ruling party. The latter bear relative costs if they do *not* vote Yes. In both cases utilities are normalized in such a way that voting No and non-acceptance of the proposal is evaluated as 0. Members of the parliament are expected to evaluate proposals under the aspect of welfare for the people. Then the absolute value of their positive or negative benefits G_i which express these evaluations should be much larger than their individual costs of voting. But we do not assume this explicitly.

We distinguish four cases. If $0 < c_i < G_i$, player i wants the proposal to be accepted without voting approvingly. If $c_i < 0 < G_i$ or $c_i < G_i < 0$, i has the dominant strategy to vote Yes. If $0 < G_i < c_i$ or $G_i < 0 < c_i$, i has the dominant strategy to vote No. If $G_i < c_i < 0$, then i wants to free-ride on the No votes of others.

[6] Negligible players’ decisions are never decisive and we can therefore restrict our attention to voting games without negligible players.

Lemma 1:

(i) Voters with dominant strategies vote according to their costs of voting. Voters with negative costs vote Yes, voters with positive costs vote No.

(ii) If voters with dominant strategies determine the vote and if all other players know this, then all players vote according to their costs.

(Without proof)

In the following we neglect voters with dominant strategies after taking their decisions into account.

Definition 2: We define player sets N^+ and N^-. For all $i \in N^+$ we have $0 < c_i < G_i$ and for all $i \in N^-$ we have $G_i < c_i < 0$. The number of players in the two sets are n^+ and n^-. All other players are assumed to follow their dominant strategies. The game with the player set $N = N^+ + N^-$ is called the *reduced simultaneous or sequential voting game.* A *narrow vote* is a reduced simultaneous voting game with $n^+ > 0$, $n^- > 0$.

Assumption 2: For equal weight voting games we assume that k, n^-, and n^+ are common knowledge. In the general case, we assume that $\mathcal{H}$, N^-, and N^+ are common knowledge.

There are several papers in the literature where lobbyists try to influence costs and benefits. While Schnakenberg (2016) describes lobbying by attempts to provide selected information which may influence G_i in our model, the bulk of the literature is about bribing which decreases (increases) c_i if the lobbyist wants to buy a Yes (No) vote. Beginning with Sneyder (1991) there are spatial models of voting (with a continuum of voters) and bribing. Dal Bo (2007) proposes a model close to ours, however with $n^- = 0$ or $n^+ = 0$, cases with completely different results (see below). He allows his lobbyist to offer a “tricky” bribing contract, namely paying a very high price for a vote which turns out to be pivotal and paying very little otherwise. Thus the lobbyist can buy, for almost nothing, a majority even if all voters have opposite preferences. Dekel et al. (2009) describe an auction with two lobbyists who compete by increasing bribe offers. Contrary to our model, however, Dekel et al. (2009)

assume that voters are only interested in costs and do not care about the result of voting (i.e. all G_i=0).

Groseclose and Milyo (2010, 2013) assume equal weight majority voting with an odd number of voters. Voting takes place in two rounds, first between alternatives *a* and *b* and then (in an undefined voting procedure) between the winner of the first vote and status quo. As status quo always wins against *a* and loses against *b* the essential vote in the first stage is between *b* and the status quo. The utility function which Groseclose and Milyo (2010, 2013) assume is only seemingly more general but essentially the same as R_i in Assumption 1. They prove Corollary 1 below and, for equal weight voting, Proposition 2 below.

II.1 Simultaneous voting

Let us first characterize voting games with unilateral interests.

Proposition 1: Let us assume $n^- = 0$.

(i) There are as many pure strategy equilibria with the acceptance of the proposal as there are minimal supporting sets. All $i \in S \in \mathcal{H}$, with $S =$ minimal set, vote Yes in such an equilibrium and all other players vote No**.**

(ii) If $\{i\} \notin \mathcal{H}$ for all i, then zero contributions by all i is the only pure strategy equilibrium without the acceptance of the proposal; otherwise no such equilibrium exists**.**

Proof: In both cases no player can gain from changing his decision. In an equilibrium without the acceptance of the proposal no player would incur costs by voting Yes.

Respective equilibria exist for $n^+ = 0$. If all players are not negligible then, except in the case $\mathcal{H} = \{N\}$, $\mathcal{H}$ contains more than one minimal set. This makes coordination of the players extremely difficult and the question arises whether completely mixed strategy equilibria (in particular when they are unique or Pareto-ranked) are more plausible candidates for equilibrium selection. In equal weight voting games with $n^- = 0$ or $n^+ = 0$ there are $m = \binom{n}{k}$ minimal sets and *m* or, for k>1, *m+1* pure strategy equilibria.

Lemma 2 *(necessary conditions for pure strategy equilibria)*: Let $S^* \subset N$ denote the set of players who vote Yes in a pure strategy equilibrium.

(i) If the proposal is accepted then $S^* \in \mathcal{H}$, $N^- \subset S^*$, $S^* - \{i\} \in \mathcal{H}$ for $i \in N^-$, and $S^* - \{j\} \notin \mathcal{H}$ for $j \in S^* - N^-$.

(ii) If the proposal is rejected then $S^* \notin \mathcal{H}$, $S^* \subset N^-$, $\{j\} \cup S^* \notin \mathcal{H}$ for $j \in N^+$, and $\{i\} \cup S^* \in \mathcal{H}$ for all $i \in N^- - S^*$.

Proof: (i) When the proposal is accepted, every player $i \in N^-$ is, because of her negative costs, better off if she votes Yes, i.e., $N^- \subset S^*$. Every player $i \in N^-$ would, however, withdraw her support if she were a pivot player; $S^* - \{i\} \in \mathcal{H}$ expresses that she is not. Every player $j \in S^* - N^-$ would, because of her positive costs, withdraw her support if she is not a pivot player; $S^* - \{j\} \notin \mathcal{H}$ expresses that she is.
(ii) In a pure strategy equilibrium without the acceptance of the proposal, support comes from $S^* \notin \mathcal{H}$. No player from N^+, because of her positive costs, would support a rejected proposal, i.e., $S^* \subset N^-$, except she were a pivot player; therefore $\{j\} \cup S^* \notin \mathcal{H}$ for $j \in N^+$ is necessary. All other players from N^- do not vote Yes because otherwise the proposal would be accepted, i.e., $\{i\} \cup S^* \in \mathcal{H}$ is required for all $i \in N^- - S^*$.

If the proposal is accepted then no player from N^- is a pivot player and every Yes voting player from N^+ is, vice versa if the proposal is rejected. All players have incentives to free ride which means that players from N^- vote Yes and players from N^+ vote No. They deviate from this behavior only if they are pivot players. Therefore the set of Yes-voters is at least N^- in (i) and the set of No-voters is at least N^+ in (ii). In the following we characterize cases where N^- is the only possible equilibrium set of Yes-voters or where no pure strategy equilibrium exists.

Definition 3: Player j is said to be *replaceable*[7] by i if, for every $S \subset N - \{i,j\}$ $S \notin \mathcal{H}$, and $S \cup \{j\} \in \mathcal{H}$, also $S \cup \{i\} \in \mathcal{H}$ applies. i and j are said to be *mutually replaceable* if i is replaceable by j and j is replaceable by i.

[7] Isbell (1958) uses the same definition but says that “i is as least as desirable as j”. In the literature on power indices desirability is used to characterize local monotonicity, i.e., if i is as least as desirable as j then i’s power index is not lower than j’s index (Freixas and Gambarelli 1997). In simple cooperative

Note that, formally, the definition applies also to the unanimity case $\mathcal{H} = \{N\}$, because $S \subset N - \{i,j\}$ and $S \cup \{j\} \in \mathcal{H}$ never applies. All players in equal weight voting games are mutually replaceable and if all players are mutually replaceable the game is an equal weight voting game (Bolle, 2015). In general voting games, there may be no replaceability relations between a pair of players[8] or they may be one-sided or mutual. Examples for one-sided replaceability are the tie-breaking vote of a chairman or the different weights of shareholders of a firm.

Proposition 2: Let $S^* \subset N$ denote the set of players who vote Yes in a pure strategy equilibrium.

(i) If $N^- - \{i\} \in \mathcal{H}$ for all $i \in N^-$ then $S^* = N^-$ describes the unique pure strategy equilibrium *with* the acceptance of the proposal. If, in addition, every $j \in N^-$ is replaceable by a player $i(j) \in N^+$ then no pure strategy equilibrium *without* the acceptance of the proposal exists.

(ii) If $N^- \cup \{i\} \notin \mathcal{H}$ for all $i \in N^+$ then $S^* = N^-$ describes the unique pure strategy equilibrium *without* the acceptance of the proposal. If, in addition, every $i \in N^+$ is replaceable by a player $i(j) \in N^-$ then no pure strategy equilibrium *with* the acceptance of the proposal exists.

Proof: **(i)** N^- fulfills the necessary conditions from Lemma 1(i). As no player from N^- has an incentive to withdraw his support and no player from N^+ has an incentive to vote Yes, $S^* = N^-$ describes an equilibrium. As no alternative set S^* fulfils the necessary conditions from Lemma 1, it is the unique equilibrium with the acceptance of the proposal. In an equilibrium S^* with the rejection of the proposal $N^- - S^*$ is not empty and contains pivot players. But as each of these pivot players can be replaced by a player from N^+ the necessary conditions from Lemma 1(ii) do not apply. **(ii)** N^- is the only set which fulfills the conditions of Lemma 1 (ii). As no player from N^- has an incentive to withdraw his support and no player from N^+ has an incentive to support the proposal, $S^* = N^-$ describes the unique equilibrium with the rejection of the proposal. An equilibrium S^* with the acceptance of the proposal has to contain

games (defined by a binary characteristic function), mutually replaceable players are called symmetric; in our voting games they can be asymmetric because of their different costs and benefits.

[8] Assume four countries (1, 2, 3, 4) with weights (2, 4, 1, 5) and populations (40, 20, 50, 10). If acceptance of a proposal requires aggregate weights of Yes-voters of at least 6 and an aggregate population of at least 60, then the only minimal supportive sets of countries are {1,2} and {3,4}. No player is replaceable by another player.

additional pivot players from N^+ and thus does not fulfill the necessary conditions from Lemma 1 (i).

If all players are mutually replaceable (have equal weights) then Proposition 2 is simplified a lot.

Corollary 1 (Groseclose and Milyo, 2010)**:** Let S^* denote the set of players who vote Yes. In a narrow vote with equal weights and $n^- \neq k-1 \; or \; k$, $S^* = N^-$ is the *unique* pure strategy equilibrium. There is *no* pure strategy equilibrium for $n^- = k-1 \; or \; k$.

Proof: Proposition 2.

Corollary 1 tells us that, in most cases, simultaneous voting with equal weights causes voting according to one's costs, independent of the benefits. Lemma 1 tells us that this is also true when players with dominant strategies determine the equilibrium. For a large number of voters as in national parliaments the exceptions $n^- = 0 \; or \; k-1 \; or \; k \; or \; n$ seem to be negligible; for small committees this is not true. The most disturbing attribute of this result is, in addition to its "preference falsification", its often implied "unintended" result. I assume that the acceptance rule of a voting system expresses the intention of the rule makers under sincere voting.

Definition 4: A proposal is said to have a *true majority* if $N^+ \in \mathcal{H}$, i.e., for equal weight voting, $|N^+| \geq k$.

Corollary 2: In a narrow vote, the result of equilibrium play is contrary to the true majority decision if $n^- < k \leq n^+$ or $n^+ < k \leq n^-$, i.e. if k is between n^+ and n^-.

Proof: Corollary 1.

The equilibrium is comparable with the zero contribution equilibrium of linear public good games where all players (unsuccessfully) try to free ride on the contributions of others. While, however, the free rider strategies in linear public good games are dominant strategies with the consequence that no other equilibrium exists, in Corollary 1uniqueness applies only with respect to the set of pure strategy equilibria. There is a large number of additional mixed strategy and pure/mixed strategy equilibria. Nonetheless, this result is important because unique pure strategy equilibria are prominent candidates for equilibrium selection.

Definition 5: Voters have weights $x_i > 0$. We define $v(S) = \sum_{i \in S} x_i$, $x^- = v(N^-)$, $m^- = \max_{i \in N^-}\{x_i\}$, $m^+ = \max_{i \in N^+}\{x_i\}$. A voting game (narrow vote) is called a *(narrow) weighted voting game* if $\mathcal{H} = \{S: v(S) \geq k\}$.

Corollary 3: Let us assume a narrow weighted voting game with $m^- \leq m^+$.

(i) If $x^- - m^- \geq k$, then $S^* = N^-$ describes the unique pure strategy equilibrium *with* the acceptance of the proposal. No pure strategy equilibrium without the acceptance of the proposal exists.

(ii) If $x^- + m^+ < k$ then $S^* = N^-$ describes the unique pure strategy equilibrium *without* the acceptance of the proposal. Additional equilibria with the acceptance of the proposal may exist only if $m^- < m^+$.

(iii) If $m^- = m^+ = m$ then no pure strategy equilibrium exists for $x^- - m < k \leq x^- + m$.

(iv) In the case $m^- > m^+$, no additional equilibria with the acceptance of the proposal exist but, possibly, equilibria without the acceptance of the proposal.

Proof: Proposition 2.

$m^- = m^+$ has almost the same implications as equal weight games, with the only difference that the threshold region without pure strategy equilibria is larger. For $m^- \neq m^+$, however, multiple pure strategy equilibria are possible.

Example 1 *(Chairman with tie breaking power):* Assume that the chairman (player 1) of a committee has tie-breaking power, i.e. a vote with k Yes-votes is accepted, a vote with k-2 Yes-votes is rejected and, if there are k-1 Yes-votes, then the chairman decides. We describe this case by giving the chairman a weight of 2 and all other members a weight of 1. In this example, Corollary 1 is "almost" confirmed. Let us assume that the chairman is from N^+, i.e. $m^- = 1$, $m^+ = 2$, and $x^- = n^-$. Corollary 3 implies that, for $n^- \geq k+1$, $S^* = N^-$ is the unique pure strategy equilibrium. For $n^- < k-2$, $S^* = N^-$ is the unique pure strategy equilibrium without the acceptance of the proposal. In this example, no equilibria with $n^- < k-2$ and the acceptance of the proposal exists because it has to be supported by at least one pivot player from N^+ with the weight 1 who would not support the proposal if $v(S^*) > k$. But then one of the players from N^- would withdraw his support. For $n^- = k-2$ and $n^- = k$ no pure strategy equilibrium exists. The only real difference to Corollary 1 is that, in the case $n^- = k - 1$, $S^* = N^- \cup \{1\}$ is an equilibrium with the acceptance of the proposal.

Example 2 *(UN Security Council):* There are five permanent members of the UN Security Council and 10 non-permanent members. The council accepts a proposal with a supermajority of 9 votes, but all permanent members have veto power. If one of the permanent members has the dominant strategy of rejecting the proposal it will be rejected. Let us instead assume that no member has a dominant strategy – the intermediate cases are similarly analyzed. The voting regime is equivalent to a weighted voting game with the permanent members having the weight[9] 10, the non-permanent members having the weight 1, and the threshold for the acceptance of the vote being 54. The analysis with the application of Corollary 3 is relegated to the appendix; the results are presented in Table 1.

Case	$m^- = 1, m^+ = 10$	$m^- = m^+ = 10$	$m^- = 10, m^+ = 1$
Equilibrium	$1 \leq x^- \leq 10$	$10 \leq x^- \leq 50$	$50 \leq x^- \leq 59$
$S^* = N^- \notin \mathcal{H}$	$1 \leq x^- \leq 10$	$10 \leq x^- \leq 44$	$50 \leq x^- \leq 53$
$N^- \subset S^* \in \mathcal{H}$	$4 \leq x^- \leq 10$	–	–
$S^* \subset N^- \notin \mathcal{H}$	–	–	$54 \leq x^- \leq 59$ §
No equilibrium	–	$45 \leq x^- \leq 50$	–

Table 1: Unique and multiple (§) equilibria in Example 2. The shaded cells are not derived from Corollary 3.

Also Example 2 seems to essentially confirm Corollary 1 because, in most cases, voting reflects costs and not true preferences. There are three important differences. First, if all members of the security council are from N^- (N^+) then we can have 2 (5) pure strategy equilibria. Second, if at least one veto player wants the proposal to be rejected then, mostly, it is rejected. As this is intended because $N^+ \notin \mathcal{H}$, we can characterize simultaneous voting as relatively successful concerning this goal. Third, however, a proposal should be accepted if $x^- \leq 6$, but this result is not supported by a unique pure strategy equilibrium; for $1 \leq x^- \leq 3$, even the contrary result is supported.

[9] Every weight w > 6 of the veto players and the threshold k=5w+4 describes voting in the Security Council.

Examples 1 and 2 describe extreme cases. In Example 1, only one player has a double weight. In Example 2 five players have so much weight that their Yes vote is necessary. Can a “medium case” have completely different implications?

Example 3 *(EEC with proposals from the commission):* The EU started in 1958 as the European Economic Community (EEC) with six member states and a decision structure where France, West Germany, and Italy had four votes, Belgium and the Netherlands had two, and Luxemburg had one. Acts of the Council, if proposed by the EEC Commission, required for their adoption twelve votes. Luxemburg is negligible in these cases because it is never member of a minimal supportive set. Therefore we analyze a voting game between the three large players and the two small players (still with k=12, all x^- are even). We restrict the analysis to the case where there are no players with dominant strategies. The analysis is as outlined in the appendix for Example 2 and the results are presented in Table 2. Equilibria which are not derived in Corollary 3 exist for $x^- = m^- = 2$ where the three large players and the small player from N^- vote Yes and for $x^- = 12$, $m^+ = 2$ where two of the three large players vote Yes and the other players vote No.

Case Equilibrium	$m^- = 2,\ m^+ = 4$ $2 \leq x^- \leq 4$	$m^- = m^+ = 4$ $4 \leq x^- \leq 12$	$m^- = 4, m^+ = 2$ $12 \leq x^- \leq 14$
$S^* = N^- \notin \mathcal{H}$	$2 \leq x^- \leq 4$	$4 \leq x^- \leq 6$	–
$N^- \subset S^* \in \mathcal{H}$	$x^- = 2$	–	–
$S^* \subset N^- \notin \mathcal{H}$	–	–	$x^- = 12^{\S}$
No equilibrium	–	$8 \leq x^- \leq 12$	$x^- = 14$

Table 2: Unique and multiple (§) equilibria in Example 2. The shaded cells are not derived from Corollary 3.

Let us summarize the results from Table 2. The true majority prevails for $x^- = 6$ where a proposal is rejected in a unique pure strategy equilibrium. In all other cases equilibria are multiple or non-existent or do not support the true majority. I think this had never been the intention of the founding fathers of EEC.

Acts of the Council, if not proposed by the Commission, required for their adoption twelve votes from at least four member states. In this case, Luxemburg is no longer negligible and voting cannot be analyzed with Corollary 3. The EEC decision rules changed with the growth of the community. In its actual form, according to the Treaty of Nice, acts from the EU council are required to be supported by a minimum number of countries with a minimum number of weighted votes and a minimum total population. In 2017, according to the Treaty of Lisbon, the triple majority is reduced to a double majority. Then, a successful vote in the EU Council requires a qualified majority of countries (55%) comprising at least 15 of them, if acting on a proposal from the Commission or from the High Representative, or else 72%, and a qualified majority of population (65%). In a transition period until 2017, a large number of exceptions apply.

All versions of multiple majorities in EU decision making as well as in the ratification of the Kyoto protocol[10] and other examples have a common attribute: the players are weakly Pareto-ranked with respect to the indices which define necessary majorities. In a country count all countries have the same weight and the order of voting weights as in the Treaty of Nice follows the order of population sizes. Therefore also the replaceability relations of countries constitute a weak order. In N^+ as well as in N^- there are countries i_+ and i_- which can replace all other counties in their set, comparable with the voters with the highest weights in Corollary 3. So, in these cases we can formulate a Corollary comparable with Corollary 3 where we substitute $m^- \leq m^+$ by "i_- is replaceable by i_+" and $x^- - m^- \geq k$ from (i) by $N^- - \{\mathrm{i}_-\} \,\epsilon \mathcal{H}$, etc..

If the multiple majority structure does not provide a weak replaceability order of voters (for an example see footnote 8) it seems difficult to characterize equilibria beyond Lemma 2 and Proposition 2.

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II.2 Sequential voting

The most investigated voting procedure in the US Senate is a roll call vote where the members are required, in alphabetic order, to vote either "yea" or "nay" and where abstention is possible in principle but usually not applied. Votes taking place on the same day, have practically always the same number of "not voting" senators[11] which

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10 Article 25 of the Protocol specifies that the Protocol enters into force "on the ninetieth day after the date on which not less than 55 Parties to the Convention, incorporating Parties … which accounted in total for at least 55% of the total carbon dioxide emissions for 1990 …, have deposited their instruments of ratification, acceptance, approval or accession."

11 See http://www.senate.gov/legislative/LIS/roll_call_lists/vote_menu_114_1.htm

is most plausibly interpreted as this number of senators being absent. So, abstention which is disregarded in our model, does not pose a major problem in this example. But roll call votes are sequential and not simultaneous votes. We may argue that, facing a fast sequence of 100 or almost 100 votes, senators have decided on their vote in advance. Therefore not only abstentions but also dynamics of voting are neglected in econometric work on roll call voting (cf. Clinton et al., 2004). Nonetheless let us derive the subgame perfect equilibrium of the sequential voting game.

Let us assume that the order of voters is (player 1, player 2, …, player n). Again we disregard all players with dominant strategies after taking their decisions into account so that the number of remaining necessary votes for the passing of the proposal is again described by $\mathcal{H}$. The game of the remaining players consists of a sequence of subgames which are essentially described by

$$S_i^* = (S^* \cap \{1, 2, \dots, i-1\}) \cup (\{i+1, \dots, n\} \cap N^+)$$

where S^* is defined as the set of players who vote Yes. Player i knows S_i^*; she knows who has voted Yes, namely $S^* \cap \{1, 2, \dots, i-1\}$, and she knows who of the remaining players wants the proposal to be accepted, namely $\{i+1, \dots, n\} \cap N^+$. If $S_i^* \cup \{i\} \in \mathcal{H}$, then player i and the remaining players in N^+ can enforce the acceptance of the proposal, if $S_i^* \notin \mathcal{H}$, player i and the remaining players in N^- can enforce rejection.

Lemma 3: The reduced sequential voting game has a unique equilibrium S^*.

(i) $i \in N^-$ votes No if she is a pivot player with respect to S_i^*; otherwise $i \in N^-$ votes Yes.

(ii) $i \in N^+$ votes Yes if she is a pivot player with respect to S_i^*; otherwise $i \in N^+$ votes No.

Proof: The proof is by backward induction. Apparently, player n will stick to the rules (i) and (ii). Then the proposal will be accepted if and only if $S_n^* \in \mathcal{H}$. Let us now assume that the proposal will be accepted if and only if $S_{i+1}^* \in \mathcal{H}$. Then player $i \in N^+$ will induce $S_{i+1}^* \in \mathcal{H}$ if he can. If he is pivotal with respect to S_i^* he must vote Yes, otherwise he saves costs and votes No. If $i \in N^-$ is pivotal with respect to S_i^* he must vote No, otherwise he incurs negative costs and votes Yes. So, (i) and (ii) apply also for player i.

In other words, players vote in accordance with their voting costs except if they are pivotal under the assumption that all following voters from N^+ vote Yes and all following players from N^- vote No.

Proposition 2:

(i) If $N^+ \notin \mathcal{H}$ then $S^* \subset N^-$ and the proposal is rejected.

(ii) If $N^+ \in \mathcal{H}$ then $N^- \subset S^*$ and the proposal is accepted.

Proof: Lemma 3.

For the case of equal weight games, Proposition 2 has been proved by Groseclose and Milyo (2013). A consequence of Proposition 2 is that, for the *result* of the vote, the order of voters in irrelevant. Individual votes, however, depend crucially on the order. As an example, take an equal weight voting game with $k = n^+ = n^-$. If the first k players are from N^+ all voters vote “Yes”, the first k because they must vote Yes in order to guarantee the acceptance of the proposal, the second k in order to incur the negative costs of voting. If the first k voters are from N^- they vote “Yes” because they cannot prevent the acceptance of the proposal, the second k vote “No” because they need not incur the positive costs of voting “Yes”.

So, while individual voting depends on the order of votes, the “sincerity of the result” is guaranteed under arbitrary orders of voters and under general decision rules. Note that the characterization $S^* \subset N^-$ if the proposal is rejected and $N^- \subset S^*$ if it is accepted also applies for simultaneous voting (Lemma 2). Nonetheless, under simultaneous voting the true majority need not win the vote.

II.3 Sequential decisions by a Lower and Upper House

In many democracies certain decisions by parliament have to be confirmed by an “upper house” (House of Lords in UK, Senate in USA, Bundesrat in Germany) or by a powerful president. In the upper house the vote is as described above; but in the lower house members take into account that there will be an additional vote if the proposal passes the lower house. Members of the lower house may be sufficiently informed about the voters in the upper house so that they can conclude how they will vote. If the proposal will not be confirmed then all members of the lower house will vote according to their costs, independent of whether they vote simultaneously or sequentially. If the proposal will be confirmed they will vote as if no upper house exists.

If the members of the lower house are not sufficiently informed about the upper house then they may estimate probabilities p_i that the proposal will pass the upper house. If these p_i are common knowledge among them then they face a problem as described above with the same c_i but with $G_i^{'} = p_i G_i$. This problem differs from the original problem only if, for some i, $\left|G_i^{'}\right| < |c_i| < |G_i|$, i.e. some additional players are endowed with dominant strategies. For simultaneous equal weight voting games (Lemma 1 and Corollary 1) there is essentially no difference to the original game, except if we no longer have a narrow vote of if we now have $n^- = k \; or \; k-1$. In a parliament with many members this may be unlikely. The result of sequential voting, however, may change as true preferences may change and therefore also the majority under true preferences may change.

III. Conclusion

An economic model of voting is suggested, which assumes Yes-No voting and arbitrary decision rules based on votes. Three crucial assumptions are that there are conflicting interests ($n^+ > 0,\; n^- > 0$), that abstentions are not allowed or not effective, and that there is complete information about qualitative interests (N^+, N^-) and the decision rule ($\mathcal{H}$). I think these assumptions are relatively weak.

First, important issues are usually supported or fought across party boarders in parliament or, with a great diversity of interests in form of costs and benefits in shareholder meetings and international organisations. Therefore the assumption $n^+ > 0$, $n^- > 0$ is not too demanding. Of course, there are situations where $n^+ = 0$ or $n^- = 0$. In parliament, preferences of the ruling party and the opposition need not even be conflicting: Almost all members of a parliament want to free-ride on the positive votes of others when the issue is a legislative pay rise. In this case, costs of voting approvingly originate in the public outrage about the self-service mentality of the parliament. Simultaneous voting games with $n^+ = 0$ or $n^- = 0$ have a plethora of asymmetric pure strategy equilibria which all seem to be less plausible than a completely mixed strategy equilibrium. Although such a structure applies for the "classical voting model", which has been proposed by Downs (1957), I think such cases are exceptions and not the rule. Downs (1957) and others have assumed costs of casting a vote, not costs of supporting or rejecting a proposal. Therefore that model is mainly used for discussing the "paradox of voting", namely that many people vote in large elections where their influence is negligible.

More critical is, second, that abstentions are assumed to be not possible or not effective because they have the same effect as voting against a proposal. This is regularly the case when an absolute quota of all members is required or at least an absolute quota (mostly majority) of the members present. Examples for absolute quotas are often "important" decisions. The German chancellor, for example, can be dropped in a vote of no confidence by the parliament only with an absolute majority of its members. Other examples are jury decisions. In Scotland 8 of 15 jury members are necessary for a *verdict*, in England 10 of 12, and, in the USA, unanimity is required. Roll call votes in the American Congress are an example where abstention is not used. (See last section.) A weak excuse for disregarding abstention is that almost all theoretical investigations on voting assume binary decisions[12]. Note, however, that abstentions will cause severe difficulties only in the case of simultaneous voting, while they will hardly change the line of argument in sequential voting.

Third, we have assumed common knowledge of k, n^-, and n^+ (for equal weights) and otherwise of $\mathcal{H}$, N^-, and N^+. Common knowledge is mostly a bold assumption but, in our model, it is not too demanding. Voters need not know the exact values of individual costs and benefits. For members of the parliament the magnitude of benefits should mostly exceed the magnitude of costs and the sign of costs are defined mostly by party membership. Then their membership to one of the four sets "players with the dominant strategy Yes", "players with the dominant strategy No", N^-, and N^+ is determined by the sign of the benefits G_i, which often is known from

[12] An exception is Battaglini (2005).

publicly available information. The interests of countries in international organizations are even more public than the interests of individuals in parliament or committees, in particular as no quantitative measures are necessary but only estimates of relative magnitudes.

The sequential and the simultaneous voting game have completely different results. In the *sequential* game, the strict condition for acceptance is that the set of voters with preferences for acceptance would win when voting sincerely: *The true majority decides the vote*. They do vote sincerely, however, only if the acceptance of the proposal is endangered.

Under *simultaneous voting with equal weights and a large number of voters* unique pure strategy equilibria mostly exist. *Voters decide according to their costs of voting "Yes"*. Therefore in parliamentary votes, party whips mostly succeed. In small committees and/or with unequal weights (shareholder decisions) or even multiple necessary majorities (EU decision making), however, pure strategy equilibria of simultaneous voting are often non-existent or non-unique. The remaining unique pure strategy equilibria support true majorities more or less. This disastrous evaluation of the "standard procedure" simultaneous voting may motivate search for its hidden advantages.

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Appendix: The derivation of the results in Table 1

First note that Corollary 3 (i) is never applicable. Let us now distinguish three cases. *Case 1:* All veto players are in N^+ which implies $x^- \leq 10$, $m^- = 1$, $m^+ = 10$. Corollary 3 (ii) is therefore always applicable. There is a unique pure strategy equilibrium without the acceptance of the proposal where all players from N^- vote Yes and the other players vote No. But an additional equilibrium with the acceptance of the proposal exists if $x^- \geq 4$, namely all veto players and the players from N^- voting Yes and the remaining non-veto players from N^+ voting No. No other equilibria exist. *Case 2:* There are veto players in N^- as well as in N^+ which implies $m^- = m^+ = 10$. Corollary 3 (ii) implies that there is a unique pure strategy equilibrium if $x^- < 44$, i.e. if there is one veto player and at least 7 non-veto players in N^+ or at least two veto players in N^+. In this equilibrium all players from N^- vote Yes and the other players vote No; i.e., the proposal is rejected. Otherwise no pure strategy equilibrium exists. *Case 3:* All veto players are in N^- which implies $x^- \geq 50$. Corollary 3 (ii)/(iv) implies a unique pure strategy equilibrium for $x^- < 53$, i.e. if at most two non-veto player are from N^-. Corollary 3 does not apply to the equilibria without acceptance of the proposal for $50 \leq x^- \leq 59$. For $54 \leq x^-$ we have five equilibria where one of the five veto-players and all players from N^+ vote No and all remaining players from N^- vote Yes.