Over- and Under-Investment According to Different Benchmarks

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Abstract. In a two-stage oligopoly, with investment in the first stage and quantity or price competition in the second stage, there is a kind of Folk Theorem: We find (i) over-investment if the goods are substitutes and competition is in strategic substitutes, (ii) under-investment if we have either complements instead of substitutes or strategic complements instead of strategic substitutes, and (iii) again over-investment if both attributes change. The existing literature, however, lacks a proof of this theorem and, in particular, it lacks a systematic comparison of the different benchmarks for over- and under-investment. A "naive" benchmark is the efficient investment with respect to the subgame perfect (closed loop) equilibrium quantities. Alternative benchmarks (which are more often proposed) are the open loop equilibrium investment or the welfare maximizing investment. The chosen benchmark is critical because the Folk Theorem applies (under certain conventional conditions) only for the naïve benchmark. The other two benchmarks require additional assumptions or the distinction of subcases.

Keywords: Oligopoly, technology choice, efficiency, under-investment, over-investment

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I. Introduction

A two-stage model with investment in the first period and quantity or price competition in the second seems to be a simple but appropriate model for long-term competition in an oligopoly. Higher investment usually decreases marginal costs, e.g. via economies of scale or via R&D for cost saving technologies. Beginning with Spence (1977), a large number of models with this structure has been discussed. The usual assumptions in the literature (with some exceptions) are “quantity competition with substitutes and strategic substitutes”. Often homogeneous goods and/or duopolies are assumed.

Sometimes interest is focussed on market entry where the model structure is slightly altered: only the incumbent invests in period 1 while the entrant chooses investment (possibly equal to 0, i.e. no entry) and price or quantity simultaneously in period 2. In particular, the seminal work of Fudenberg and Tirole (1984) and Bulow et al. (1983) has lead to the impression that the logic of such models is principally understood.

The consequences of such a model structure are often described as over- or under-investment. While Brander and Spencer (1983) evaluate investment with respect to the “naïve” benchmark[2] of whether the closed loop (subgame perfect) equilibrium quantities are produced efficiently, Fudenberg and Tirole (1984) as well as Bulow et al. (1985) propose the open loop equilibrium (where investment and prices/quantities are chosen simultaneously) as a benchmark for over- and under-investment. Their reasoning is that the open loop benchmark incorporates non-strategic investment (efficient production of the open loop equilibrium quantities) so that “over” and “under” express the consequences of strategic considerations. We could also interpret such a benchmark as the goal of a regulator who wants production to be efficient. In the energy sector, for example, inefficient production may be accompanied by unnecessarily high levels of CO_2 emissions (under-investment) or too many highly controversial nuclear power plants (over-investment). The open loop benchmark is also used by Okuno-Fujiwara and Suzumura (1993) and Murphy and Smeers (2005).

[2] This is an obvious benchmark. I call it naïve because it is difficult to see how and why regulation could and should implement this benchmark.

Many authors prefer to compare equilibrium investment in the two stage (closed loop) equilibrium with socially optimal investment, usually in the form of a second best welfare optimum. Second best means that only investment is regulated, but not competition in the second stage of the game. Suzumura (1992), and Okuno-Fujiwara and Suzumura (1993) apply this benchmark. In this paper, we want to concentrate on the three benchmarks mentioned above.

There are papers which use other benchmarks, namely the first best welfare optimum (Suzumura, 1992; Elberfeld, 2003; Murphy and Smeers, 2005) or cooperation among the oligopolists in the first round (Suzumura, 1992; Long and Sobeyran, 2001). Some papers are not directly concerned with under-/over-investment but instead study optimal taxes or subsidies (Besley and Suzumura, 1992; Vetter, 2007). As the above citations show, several papers use multiple benchmarks. On the other hand, there are papers which investigate the subgame perfect equilibrium (the closed loop solution) without comparisons (Tseng, 2003; Grant and Quiggin, 1996).

A number of two-stage games which seem to be different at first glance can be interpreted in the above frame work (see Shapiro, 1989). Allaz (1992) and Bolle (1993), for example, discuss the consequences of the introduction of a futures market (for oligopolies such as electricity). In this case the benchmark is the non-existence of such a market (zero investment, i.e. buying/selling of futures contracts). Other applications may suggest further benchmarks.

A common attribute of the three benchmarks which we will discuss is that there is oligopolistic competition in the second stage. Thus, it should not be too difficult to extend the investigation to the benchmark “cooperation of the oligopolists in the first stage” and to other benchmarks which share this common attribute. I think, however, such an extension would overload the paper. A general comparison with the first best welfare optimum would be more difficult because the decisions in the second stage are also involved. Discussing the first best optimum is often the same as substituting the industry with a regulated monopoly.

There are a number of extensions of the basic model: more periods of investment (e.g. Stanford, 1986; Athey and Schmutzler, 2001; Bolle and Breitmoser, 2004), spillovers in R&D (d'Aspremont and Jacquemin 1988; Kamien et al.; 1992), incomplete information (Vives, 1989; Somma, 1999), and others. We will not consider such extensions, but instead concentrate on the role of benchmarks in the basic case.

In the next section we describe the basic model and its subgame perfect equilibrium within a conventional model with minimal assumptions. In Section III, we add the (conventional) assumptions of symmetry and of stability of the second stage market and show that for the naïve benchmark, the following theorem applies:

Folk Theorem: There is (i) over-investment if the goods are substitutes and competition is in strategic substitutes, (ii) under-investment if we have either complements instead of substitutes or strategic complements instead of strategic substitutes, and (iii) again over-investment if both attributes change.

In Sections IV and V, we derive the results for the open-loop and for the (second best) welfare benchmark. Under the open loop benchmark, the Folk Theorem applies only for the case of strategic substitutes; under the welfare benchmark, it applies only for the case complements & strategic substitutes. In Section VI, a simple example is discussed which shows that for non-naïve benchmarks we indeed encounter more complicated cases. It is also shown that two extensions of the stability requirement do not affect this result.

The main part of the paper concentrates on quantity competition in the second stage. The investigation of price competition is relegated to the appendix. It implies the same qualitative results as quantity competition. Section VII reports[3] and discusses the joint results, in particular the additional distinctions which must be introduced.

II. Competition with cost functions

In principle, we may distinguish three stages of decisions. In the first stage, technology is chosen, i.e. firms invest in R&D, buy patents, sign licence contracts, purchase sites, and seek strategic partnerships. In hierarchical production processes

[3] See Table 1 in Section VII.

“make or buy” decisions are also important. These decisions influence marginal costs mainly due to the choice of technology but also because of license fees, transportation costs (and modes), and transaction costs. In the second stage, capacity is built which may influence marginal costs due to economies of scale and scope. In the third stage, we have competition in quantities, prices, or other instruments. We will simplify this structure by merging stages 2 and 3. Alternatively we could merge stages 1 and 2 or we could analyse the three-stage game. Apart from the fact that we follow the bulk of the literature, both alternatives would be more complicated.

The technologies available to firm i are described by a one-parameter family of cost functions $C_i(c_i, x_i)$, x_i = quantity produced and $c_i \in$ **R** = real number. We require

(1) $\quad \partial C_i / \partial x_i > 0, \quad \partial^2 C_i / \partial x_i^2 > 0, \quad \partial^2 C_i / \partial x_i \partial c_i > 0, \quad \partial^2 C_i / \partial c_i^2 > 0.$

The first two requirements are standard. The third requirement is that the marginal cost curves increase in c_i. It is implicitly assumed that fixed costs decrease in c_i; therefore $C_i(c_i, x_i)$ may increase or decrease in c_i. The last requirement must apply for interior cost minima. For the following, we define $x = (x_1, ..., x_n)$ and $c = (c_1, ..., c_n)$.

Game CC (competition with cost functions):

Stage 1 (Technology Choice): Firm i chooses c_i, $i = 1, ..., n$.

Stage 2 (Competition with Quantities): Firm i chooses x_i. Demand for the product of firm i is described by the inverse demand function $p_i(x)$. $i = 1, ..., n$.

We assume that interior equilibria of the game exist. We denote a subgame perfect equilibrium of game CC as $(x(c), \tilde{c})$. $x(c)$ describes the second stage equilibrium quantities for a given c. The efficient cost parameter vector c for the quantities $\tilde{x} = x(\tilde{c})$ is denoted by $\tilde{c}^*$. In this and the following section we apply the naïve benchmark, i.e. we say that cost function competition results in over-investment if $\tilde{c} < \tilde{c}^*$ and in under-investment if the opposite relations hold.

The profit of firm i is $G_i = x_i p_i(x_i) - C_i(c_i, x_i)$. An interior second-stage equilibrium of the CC game $x(c)$ satisfies

(2) $$\frac{\partial G_i}{\partial x_i} = p_i(x) + x_i \frac{\partial p_i}{\partial x_i} - \frac{\partial C_i}{\partial x_i} = 0, \quad \text{i=1,…,n} \qquad \text{and}$$

(3) $$\frac{\partial^2 G_i}{\partial x_i^2} = 2\frac{\partial p_i}{\partial x_i} + x_i \frac{\partial^2 p_i}{\partial x_i^2} - \frac{\partial^2 C_i}{\partial x_i^2} < 0 .$$

The total derivative of eq. (2) with $dx_j = 0$ for $j \neq i,k$ provides us with the slope of i's best reply function $x_i = f_i(x_{-i})$ with respect to x_k.

(4) $$\frac{dx_i}{dx_k} = -\frac{\frac{\partial p_i}{\partial x_k} + x_i \frac{\partial^2 p_i}{\partial x_i \partial x_k}}{2\frac{\partial p_i}{\partial x_i} + x_i \frac{\partial^2 p_i}{\partial x_i^2} - \frac{\partial^2 C_i}{\partial^2 x_i^2}} .$$

Because of relation (3), we have strategic substitutes (complements) if

(5) $$\frac{\partial p_i}{\partial x_k} + x_i \frac{\partial^2 p_i}{\partial x_i \partial x_k} < 0 \ (> 0)$$

applies.

Assumption:

(A1) The second stage equilibria $x(c)$ are unique differentiable functions of c.

Applying the Implicit Function Theorem, we obtain

(6) $$\left(\frac{dx_i}{dc_k}\right) = -\left(\frac{\partial^2 G_i}{\partial x_i \partial x_k}\right)^{-1} \left(\frac{\partial^2 G_i}{\partial x_i \partial c_k}\right).$$

$\left(\frac{\partial^2 G_i}{\partial x_i \partial c_k}\right)$ is a diagonal matrix with negative diagonal elements $-\partial^2 C_i / \partial x_i \partial c_i$.

$\left(\frac{\partial^2 G_i}{\partial x_i \partial x_k}\right)$ has diagonal elements $2\frac{\partial p_i}{\partial x_i} + x_i \frac{\partial^2 p_i}{\partial x_i{}^2} - \frac{\partial^2 C_i}{\partial x_i{}^2} < 0$ (because of eq. (3)) and otherwise elements $\frac{\partial p_i}{\partial x_k} + x_i \frac{\partial^2 p_i}{\partial x_i \partial x_k}$ (<0 for strategic substitutes).

Firm i's best reply to c_{-i}, the technology choice of the other firms, is derived from

(7) $$\frac{dG_i}{dc_i} = \frac{\partial G_i}{\partial c_i} + \frac{\partial G_i}{\partial x_i} \cdot \frac{dx_i}{dc_i} + \sum_{k \neq i} \frac{\partial G_i}{\partial x_k} \cdot \frac{dx_k}{dc_i} = 0.$$

As $\frac{\partial G_i}{\partial x_i} = 0$ (Stage 2), $\frac{\partial G_i}{\partial c_i} = -\frac{\partial C_i}{\partial c_i}$, and $\frac{\partial G_i}{\partial x_k} = \frac{\partial p_i}{\partial x_k} x_i$, we get

(8) $$\frac{dG_i}{dc_i} = -\frac{\partial C_i}{\partial c_i} + x_i \sum_{k \neq i} \frac{\partial p_i}{\partial x_k} \cdot \frac{dx_k}{dc_i} = 0 .$$

Contrary to eq. (8), cost efficiency requires

(9) $$\frac{\partial G_i}{\partial c_i} = -\frac{\partial C_i}{\partial c_i} = 0 .$$

For equilibrium values, the difference between eqs. (8) and (9) is

(10) $$\tilde{z}_i = x_i \sum_{k \neq i} \frac{\partial p_i}{\partial x_k} * \frac{dx_k}{dc_i}\bigg|_{\tilde{x}, \tilde{c}} .$$

The implicit function $\partial C_i / \partial c_i = \tilde{z}_i$ lies above (below) $\partial C_i / \partial c_i = 0$ if the sign of $\tilde{z}_i$ is positive (negative).

Definition: The following derivatives are evaluated at $(\tilde{x},\tilde{c})$. We set $g := +1$ if all $\partial p_i / \partial x_k > 0$, i.e. if goods are complements, and $g := -1$ if all $\partial p_i / \partial x_k < 0$, i.e. if goods are substitutes. Otherwise, we set $g := 0$. We define $h := +1$ *if all* $dx_i / dc_k > 0$ and $h := -1$ *if all* $dx_i / dc_k < 0$. Otherwise, we set $h := 0$.

Proposition 1: If $g * h = -1\,(+1)$, then $\tilde{x}$ is produced with over-investment (under-investment) according to the naïve benchmark.

Proof: Because of relations (1), the implicit functions $x_i(c_i)$ defined by $\partial C_i / \partial c_i = const$ have a negative slope. $g * h = +1\,(-1)$ means that the implicit function $\partial C_i / \partial c_i = \tilde{z}_i$ lies above (below) $\partial C_i / \partial c_i = 0$. (See Figure 1.)■

We could generalize Proposition 1 by defining g_i and h_i for every firm i separately, i.e. Proposition 1 applies also for a market where some goods are (strategic) substitutes and others are (strategic) complements.

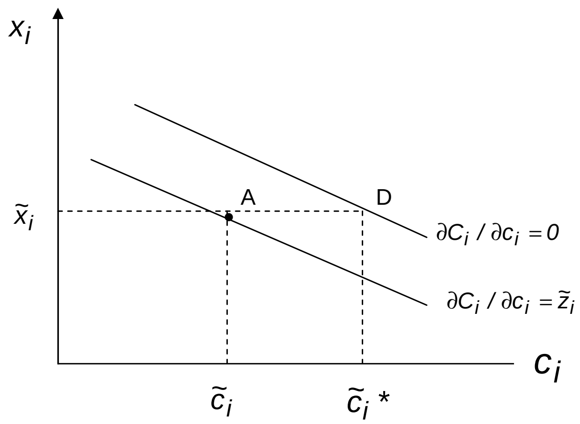

Figure 1: An equilibrium A of the CC game and efficient production of the equilibrium quantity in the case $\tilde{z}_i < 0$ $(g * h = -1)$.

The sign of h can be determined by the Implicit Function Theorem – but that requires either numerical values or special cases. In the next section, we assume symmetric and stable[4] equilibria, in which case we obtain $h = -1$ for strategic complements and $h = +1$ for strategic substitutes.

III. Symmetric and stable equilibria

Assumptions:

(A2) The game CC is symmetric and has interior symmetric equilibria $(x(c),\tilde{c})$.

(A3) The symmetric second stage equilibria of the CC game are stable.

In symmetric games, all producers can choose from the same set of technologies. For symmetric (inverse) demand functions $p_i(x)$ the variables x_j and x_k, i≠j,k, are interchangeable and we have $p_i(..., x_{i-1}, x_i,..., x_{k-1}, x_k,...) = p_k(..., x_{i-1}, x_k,..., x_{k-1}, x_i,...)$. In a symmetric equilibrium all c_i are equal, i.e. we have $c = c^s = (c_0,...,c_0)$. The second stage equilibrium $x(c^s)$ shows $x_1(c^s) = ... = x_n(c^s) =: x_0(c^s)$. Evaluated at these equilibrium values, $\left(\frac{\partial^2 G_i}{\partial x_i \partial x_k}\right)$ has identical diagonal elements α<0 and identical non-diagonal elements β.

In the second stage of the CC game, c is given. $x(c)$ is stable if the iterative process

(11) $$x_i^{m+1} = f_i(x_{-i}^{m}),\ i = 1,..., n\text{ , m = 1, 2, …}$$

with $f_i(x_{-i}^{m}) =$ i's best reply function, converges to $\tilde{x}(c)$ for $x^1 = (x_1^1,..., x_n^1)$ sufficiently close to $x(c)$. This convergence requires (in the

[4] Stability of equilibria is discussed, for example, in Puu and Suskho (2002).

case of a symmetric equilibrium) that the matrix M of the partial derivatives of $f_i(x_{-i}{}^m)$ defined by diagonal elements 0 and identical non-diagonal elements $dx_i / dx_k = -\beta / \alpha$ has no eigenvalue λ with $|\lambda| \geq 1$. Apparently (1,…,1) is an eigenvector of M with the eigenvalue $\lambda = -(n-1)\beta / \alpha$. We conclude that stable symmetric second-stage equilibria require

(12) $$(n-1)\left|\frac{\beta}{\alpha}\right| < 1 \ .$$

Let us now determine the sign of $dx_k(c_1,...,c_n)/dc_i$ which is used above to define h. In a symmetric equilibrium $\left(\frac{\partial^2 G_i}{\partial x_i \partial x_k}\right)^{-1}$ has constant non-diagonal elements

(13) $$b = \frac{\beta}{(n-1)\beta^2 - \alpha^2 - (n-2)\alpha\beta}$$

and constant diagonal elements

(14) $$a = \frac{1-(n-1)b\beta}{\alpha} \ .$$

Lemma 1: Evaluated at symmetric and stable second stage equilibria, the following relations hold:

(i) $\frac{dx_i(c_1,...,c_n)}{dc_i} = a * \frac{\partial^2 C}{\partial c_i \partial x_i} < \frac{1}{\alpha} * \frac{\partial^2 C}{\partial c_i \partial x_i}$

(ii) $\frac{dx_i(c_1,...,c_n)}{dc_k} = b * \frac{\partial^2 C}{\partial c_i \partial x_i} > 0 \ (<0)$ for strategic substitutes (complements)

(iii) $\frac{dx_0(c_0,...,c_0)}{dc_0} = (a+(n-1)b) * \frac{\partial^2 C}{\partial c_i \partial x_i} > (<) \ \frac{1}{\alpha} * \frac{\partial^2 C}{\partial c_i \partial x_i}$ for strategic substitutes (complements)

(iv) $a + (n-1)b = \frac{\beta - \alpha}{(n-1)\beta^2 - \alpha^2 - (n-2)\alpha\beta} < \left\{ \begin{matrix} nb & \text{for } b<0 \\ -(n-2)b & \text{for } b>0 \end{matrix} \right\} \leq 0 \ .$

Proof: Strategic substitutes (complements) are defined by $\beta < 0$ ($\beta > 0$). Relation (12) implies

(15) $$(n-1)\beta^2 - \alpha^2 - (n-2)\alpha\beta < 0 \ .$$

Thus eq. (13) implies $\beta b < 0$, and therefore $a < 1/\alpha < 0$. Then (i) and (ii) follow from eq. (6). (iii) follows from $\frac{dx_i(c_0,...,c_0)}{dc_0} = \sum_k \frac{dx_i}{dc_k} = (a+(n-1)b) * \frac{\partial^2 C}{\partial c_i \partial x_i}$ and $a + (n-1)b = \frac{1}{\alpha} - (n-1)b(\frac{\beta}{\alpha} - 1) > (<) \frac{1}{\alpha}$ for $\beta < 0$ ($\beta > 0$) which implies $b > 0$ ($b < 0$). The first part of (iv) follows from eqs. (13) and (14), the second part is implied by relation (12). ■

Lemma 1 (ii) informs us that, under the above assumptions, $h = +1$ for strategic substitutes and $h = -1$ for strategic complements. The other parts of Lemma 1 are utilised in Sections IV and V.

Corollary: The Folk Theorem described in the Introduction applies for the naïve benchmark.
Proof: Proposition 1 and Lemma 1 (ii).

IV. The open loop benchmark

We now confront the subgame perfect (closed loop) equilibrium $(x(c), \tilde{c})$ of Game CC with the open loop equilibrium or, equivalently, with the equilibrium (x^*, c^*) of a one-stage benchmark game EC (efficient costs) where technologies and quantities are chosen simultaneously.

Assumption:

(A4) Game EC has a unique and symmetric equilibrium (x^*, c^*).

Lemma 2: $\dfrac{dx_0(c_0,...,c_0)}{dc_0}$, the slope of the function which describes the second stage equilibrium quantities with identical $c_i = c_0$, is larger than the slope of $\partial C_0 / \partial c_0 = const$ if and only if

$$\text{(C1+)} \qquad \frac{\partial C_0^2}{\partial c_0^2} > \frac{\alpha - \beta}{(n-1)\beta^2 - \alpha^2 - (n-2)\alpha\beta}\left(\frac{\partial^2 C_0}{\partial c_0 \partial x_0}\right)^2 .$$

Proof:
Taking into account Lemma 3 (iv), relation (C1+) is equivalent to

$$\frac{dx_0}{dc_0} = (a + (n-1)b)\frac{\partial C_0}{\partial c_0 \partial x_0} > -\frac{\partial^2 C_0 / \partial c_0^2}{\partial^2 C_0 / \partial c_0 \partial x_0} = slope\, of\ \frac{\partial C_0}{\partial c_0} = const\ . \ ■$$

Condition (C1+) is decisive in the case of strategic complements. Imagine that all firms increase their investment by one unit. One effect is the decrease of marginal costs, the other is increased market supply which again increases marginal costs. Condition (C1+) requires that the net effect is deceased marginal costs. (C1-) describes the opposite relation, i.e. “larger” is substituted by “smaller” in (C1+).

Proposition 2: Let us assume (A 1), (A 2), (A 3), (A 4). Evaluations with respect to the distinction between (strategic) substitutes or complements take place at (x^*,c^*).

(i) *Strategic substitutes*: There is over-investment (under-investment) in Game CC according to the open loop benchmark if goods are substitutes (complements).

(ii) *Strategic complements*: If (C1+) applies and if goods are substitutes (complements) then there is under- (over-) investment according to the open loop benchmark. If relation (C1-) applies, we obtain the opposite result.

Proof: The symmetric equilibrium (x^*,c^*) requires eqs. (2) and (9) to hold for all i. It is also a second-stage equilibrium $x^* = x(c^*)$ of the CC game. Therefore (x^*,c^*) is determined by the intersection of the curve defined by eq. (9) with the curve $x_0(c_0,...,c_0)$. The symmetric equilibrium $(\tilde{x},\tilde{c})$ is determined by the intersection of $x_0(c_0,...,c_0)$ with the curve defined by eq. (8) where $\tilde{z}$ assumes the equilibrium

value. The slopes of $x_0(c_0,...,c_0)$ and $\partial C_0 / \partial c_0 = const$ are calculated in Lemma 1 (iii) and Lemma 2.

(i) In the benchmark game EC, the second order condition for best replies is a negative definite Hessian of the profit function, i.e. $\partial^2 G_i / \partial c_i^2 * \partial^2 G_i / \partial x_i^2 - (\partial^2 G_i / \partial c_i \partial x_i)^2 = \partial^2 C / \partial c_i^2 * \alpha - (\partial^2 C / \partial c_i \partial x_i)^2 > 0$

which is equivalent to

$$(16) \qquad \frac{\partial^2 C(c_i,x_i)/\partial x_i \partial c_i}{\alpha} > -\frac{\partial^2 C(c_i,x_i)/\partial c_i^2}{\partial^2 C(c_i,x_i)/\partial x_i \partial c_i}.$$

For strategic substitutes, relation (16) and Lemma 1 (iii) show that, at their intersection, $x_0(c_0,...,c_0)$ has a larger slope than $\partial C_i / \partial c_i = 0$, i.e. (C1+) applies. This implies that all $\tilde{c} < c^* (> c^*)$ if $\tilde{z} < (>) 0$, i.e. for substitutes (complements). See Figure 2.

(ii) If (C1+) is fulfilled, then we can argue as under (i). When the relation of slopes changes, we obtain the opposite results.■

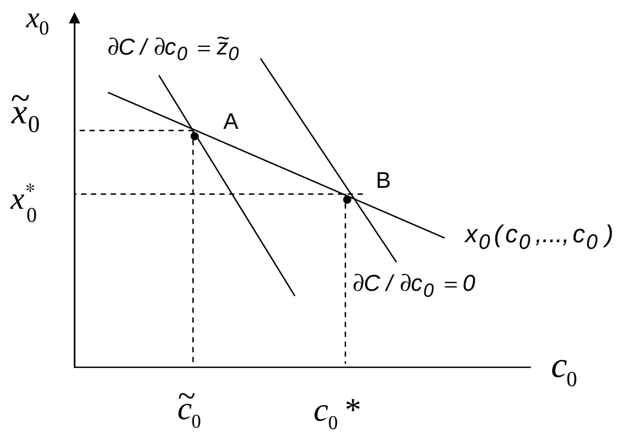

Figure 2: The equilibrium A of the CC game and the equilibrium B of the EC game in the case of substitutes and strategic substitutes.

Remark: $(x(c),\tilde{c})$ need not be unique as $\tilde{z}$ might assume different equilibrium values. Even for the same $\tilde{z}$ there can be different $(x(c),\tilde{c})$ if evaluations with respect to the differentiation between (strategic) substitutes or complements are not the same for all symmetric equilibria (in which case the relation of the slopes may change for values differing from c^*). But all the $\tilde{c}$ must be smaller (larger) than c^* if goods are substitutes (complements).

The example in Section VI will show that, for strategic complements, (C1+) may or may not be fulfilled. There, we will also show that additional stability assumptions cannot avoid the distinction.

V. The (second best) welfare benchmark

Welfare is measured here as the sum of consumers' and producers' surplus, i.e.

$$(17) \quad W = \sum_i \left[\int_0^{x_i(c_1,\ldots,c_n)} p_i(\xi_i, x_{-i}) d\xi_i - C_i(x_i(c), c_i) \right].$$

Let us assume that regulatory measures are introduced only with respect to investment. The second stage of the game is still an oligopoly where quantities $x_i(c)$ are chosen. An interior (second best) optimum then requires

$$(18) \quad \frac{\partial W}{\partial c_k} = \sum_i \left[\int_0^{x_i(c_1,\ldots c_n)} \sum_{j \neq i} \frac{\partial p_i}{\partial x_j} \frac{dx_j}{dc_k} d\xi_i + \frac{dx_i}{dc_k} \left(p_i - \frac{\partial C_i}{\partial x_i} \right) \right] - \frac{\partial C_k}{\partial c_k} = 0$$

for k = 1, …, n. Because of the second stage best replies eq. (2), we can substitute $p_i - \frac{\partial C_i}{\partial x_i} = -x_i \frac{\partial p_i}{\partial x_i}$.

Let us now assume:

(A5) The system of equations (18) has a unique and symmetric[5] solution $\hat{c} = (\hat{c}_0, \ldots, \hat{c}_0)$.

We say that there is under- (over-) investment with respect to the welfare benchmark if $\tilde{c}_0 > (<) \hat{c}_0$.

The equilibrium quantities implied by $\hat{c}$ are $\hat{x} = x(\hat{c})$. $x(\hat{c})$ is the same function as in the previous sections. We can now proceed as in the last section, except that eqs. (8) are substituted by eqs. (18), i.e. $\tilde{z}_i$ is substituted by

$$(19) \quad \hat{z}_i = \sum_i \left[\int_0^{\hat{x}_i(c_1,\ldots c_n)} \sum_{j \neq i} \frac{\partial p_i(\xi_i, x_{-i})}{\partial x_j} \cdot \frac{dx_j}{dc_k} d\xi_i - \frac{dx_i}{dc_k} x_i \frac{\partial p_i}{\partial x_i} \right].$$

Lemma 1 shows that $\frac{dx_j}{dc_k} = b \cdot \frac{\partial^2 C_i}{\partial x_i \partial c_i}$, $\frac{dx_k}{dc_k} = a \cdot \frac{\partial^2 C_k}{\partial x_k \partial c_k}$ with a and b defined in eq. (13) and (14). For identical c_i we get ($k \neq i$)

$$(20) \quad \hat{z}_0 = \hat{z}_i = ((n-1)b + a) \left[(n-1) \int_0^{x_i} \frac{\partial p_i(\xi_i, x_{-i})}{\partial x_k} d\xi_i - x_i \frac{\partial p_i}{\partial x_i} \right] \frac{\partial^2 C}{\partial c_i \partial x_i}.$$

For complements, $\tilde{z}_i$ and $\hat{z}_i$ are relatively easy to compare as, in eq. (20), $\frac{\partial p_i}{\partial x_k}$ and $-\frac{\partial p_i}{\partial x_i}$ have the same sign. In the case "complements and strategic substitutes" we need n>2 and a very weak additional assumption, namely

$$(A6) \quad (n-1) \int_0^{x_i} \frac{\partial p_i(\xi_i, x_{-i})}{\partial x_k} d\xi_i - x_i \frac{\partial p_i}{\partial x_i} \geq -\frac{n-1}{n-2} x_i \frac{\partial p_i}{\partial x_k} \quad for\ n > 2.$$

For substitutes, the result of the comparison depends on the question of how differentiated the goods are. In the case of substitutes and strategic substitutes, sufficient alternative conditions are:

[5] As the industry structure in two stage oligopolies often has the character of a natural monopoly or oligopoly, the second best welfare optimum may be asymmetric. This is therefore a strong assumption. On the other hand, there are often symmetric optima as the example in Section VI shows.

(C2+) $$(n-1)\int_0^{x_i} \frac{\partial p_i(\xi_i, x_{-i})}{\partial x_k} d\xi_i - x_i \frac{\partial p_i}{\partial x_i} \le 0$$

or

(C2-) $$(n-1)\int_0^{x_i} \frac{\partial p_i(\xi_i, x_{-i})}{\partial x_k} d\xi_i - x_i \frac{\partial p_i}{\partial x_i} \ge -\frac{n-1}{n-2} x_i \frac{\partial p_i}{\partial x_k} \quad \text{for } n > 2 \ .$$

(C2-) limits the homogeneity of the goods while (C2+) requires sufficient homogeneity. For n=2 and linear demand functions $p_i = 1 - x_i - \gamma x_k$, (C2+) requires $\gamma \ge 1$, i.e. homogeneous goods or "more than homogeneous" goods which may be interpreted as "network effects". With increasing n, the set of cases not covered by (C2-) and (C2+) becomes smaller.

In case of substitutes and strategic complements, sufficient alternative conditions are:

(C3+) $$(n-1)\int_0^{x_i} \frac{\partial p_i(\xi_i, x_{-i})}{\partial x_k} d\xi_i - x_i \frac{\partial p_i}{\partial x_i} \le \frac{n-1}{n} x_i \frac{\partial p_i}{\partial x_k} \ ,$$

(C3-) $$(n-1)\int_0^{x_i} \frac{\partial p_i(\xi_i, x_{-i})}{\partial x_k} d\xi_i - x_i \frac{\partial p_i}{\partial x_i} \ge 0 \ .$$

Again, (C3-) limits homogeneity while (C3+) requires enough of it.

Contrary to the last section, we do not assume equilibrium values for the evaluation of $\tilde{z}_i$ and $\hat{z}_i$, but we take both as functions of $c^s = (c_0, ..., c_0)$. In the comparisons of $\tilde{z}_i$ and $\hat{z}_i$ below, we assume the same argument c^s.

Note that, as an auxiliary device, we also assume (A4). The reason is that it is far easier to compare the slopes of $x_0 = x_0(c^s)$ and $\partial C_0 / \partial c_0 = 0$ (see relation (C1+)) than those of $x_0 = x_0(c^s)$ and $\partial C_0 / \partial c_0 = \hat{z}_0(c^s)$ (or $\partial C_0 / \partial c_0 = \tilde{z}_0(c^s)$).

Proposition 3: Let us assume that (A1) to (A6) apply.

(i) *Substitutes and strategic substitutes (for all $x(c^s)$):* If (C2+) applies, then there is over-investment in the CC game with respect to the welfare benchmark. If (C2-) applies (which also means n > 2), then there is under-investment.

(ii) *Substitutes and strategic complements (for all $x(c^s)$):* If relations (C3-) and relation (C1+) apply, then there is under-investment with respect to the welfare benchmark. If relations (C3+) and (C1+) apply, then there is over-investment. If relation (C1-) applies, we obtain the opposite results.

(iii) *Complements and strategic substitutes (for all $x(c^s)$)*: For n > 2, there is under-investment with respect to the welfare benchmark.

(iv) *Complements and strategic complements (for all $x(c^s)$)*: If relation (C1+) applies there is over-investment with respect to the welfare benchmark. If relation (C1-) applies, we get the opposite result.

Proof: *(i) and (C2-):* Eq. (20) and Lemma 2 (ii) and (iv) imply

(21) $$\hat{z}_i < (n-1) b x_i \frac{\partial p_i}{\partial x_k} = \tilde{z}_i < 0 \ .$$

For the remaining argument let us look at Figure 3. In the case of strategic substitutes, (C1+) always applies, i.e. the slope of $x_0(c^s)$ is larger than the slope of $\partial C_0 / \partial c_0 = 0$. For the unique welfare optimum $\hat{c}_0$, the slope of $x_0(c^s)$ has to be larger than the slope of $\partial C_0 / \partial c_0 = \hat{z}_0(c^s)$. Otherwise either a second solution of the system of equations (18) would exist (contradicting (A5)), or $\partial C_0 / \partial c_0 = \hat{z}_0(c^s)$ would have to enter the region of positive $\hat{z}_0$ (contradicting relation (21)). In addition, relation (21) shows that $\partial C_0 / \partial c_0 = \hat{z}_0(c^s)$ lies below $\partial C_0 / \partial c_0 = \tilde{z}_0(c^s)$. $\tilde{c}_0$ is not necessarily unique, but all $\tilde{c}_0$ must be larger than $\hat{c}_0$.

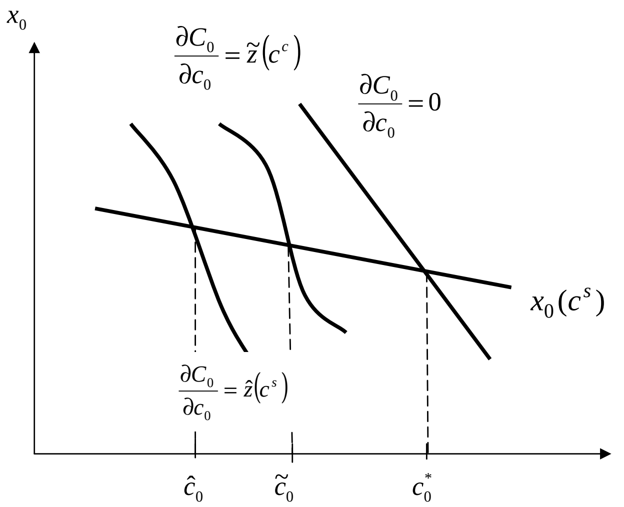

Figure 3: Under-investment under the welfare benchmark when $\hat{z}_i < \tilde{z}_i < 0$ and when (C1+) applies.

(i) and (C2+): From eq. (10) and Lemma 2 follows $\tilde{z}_i < 0$ while eq. (20) and Lemma 2 (iv) imply $\hat{z}_i > 0$. Taking this difference into account in Figure 3, we get over-investment.

(ii) and (C3-): From eqs. (10) and (20) and Lemma 2 follows $\hat{z}_i < 0$ and $\tilde{z}_i > 0$. If relation (16) applies, we can again use Figure 3 to show that under-investment results. If relation (C1-) applies, the slope of $x_0(c^s)$ is smaller than the slope of $\partial C_0 / \partial c_0 = 0$ and we obtain the opposite result.

(ii) and (C3+): In situation (ii), (C3+) implies $\hat{z}_i > \tilde{z}_i > 0$. When relation (C1+) applies, then over-investment occurs. Otherwise there is under-investment.

(iii): From eqs. (10) and (20) and Lemma 2 follows $\hat{z}_i < 0$ and $\tilde{z}_i > 0$. So we have the same situation as in (ii) and (C3+).

(iv): Because of (A6) we get

(22) $$\hat{z}_i < (n-1)bx_i \frac{\partial p_i}{\partial x_i} = \tilde{z}_i < 0 .$$

Therefore, if (C1+) applies, we are in the situation of Figure 3 again. If relation (C1-) applies, the slope of $x_0(c^s)$ is smaller than the slope of $\partial C_0 / \partial c_0 = 0$ and we obtain the opposite result. ■

VI. An example

Let us assume linear demand and constant marginal costs. Fixed costs are quadratic in c_i.

(23) $$p_i = 1 - x_i - \gamma \sum_{k \neq i} x_k .$$

(24) $$C(c_i, x_i) = \overline{C} - rc_i + \frac{s}{2} c_i^2 + c_i x_i , \quad r, s > 0 .$$

Thus we get $\alpha = -2$ and $\beta = -\gamma$, where α *and* β are defined in Section III as the diagonal and non-diagonal elements of the matrix $\left(\frac{\partial^2 G_i}{\partial x_i \partial x_k}\right)$. With the same matrix, we represent the first order conditions $\frac{\partial G_i}{\partial x_i} = 1 - x_i - \gamma \sum_{k \neq i} x_k - x_i - c_i = 0$:

(25) $$-\left(\frac{\partial^2 G_i}{\partial x_i \partial x_k}\right) x = \begin{pmatrix} 1 - c_1 \\ \ldots \\ 1 - c_n \end{pmatrix}$$

From (13) and (14) we get the unique second stage equilibrium

(26) $$x_i = -a(1 - c_i) - b \sum_{k \neq i} (1 - c_k), \quad i = 1, \ldots, n \quad \text{with}$$

$$b = \frac{\gamma}{(n-2)2\gamma + 4 - (n-1)\gamma^2} \quad \text{and } a = -\frac{1 + (n-1)b\gamma}{2} .$$

For identical $c_i = c_0$ we get

(27) $$\tilde{x}_0 = \tilde{x}_i = \left(\frac{1}{2} - (n-1)b\left(1 - \frac{\gamma}{2}\right)\right)(1 - c_0) .$$

The equations

$$\frac{\partial C(c_0, x_0)}{\partial c_0} = x_0 - r + sc_0 = \tilde{z}_0 \left(= -\frac{\gamma^2}{4-\gamma^2} x_0 \;\; for\; n = 2 \right) \tag{28}$$

are the first stage requirements of the closed loop equilibrium (with 0 instead of $\tilde{z}_0$ for the open loop equilibrium). Thus the system of equations (25) and (28) determines the closed loop equilibrium $(\tilde{x}, \tilde{c})$ and, for 0 instead of $\tilde{z}_0$, the open loop equilibrium (x^*, c^*). For an interior second best welfare optimum $(\hat{x}, \hat{c})$, in (28), $\tilde{z}_0$ has to be substituted $\hat{z}_0 \left(= -\frac{1-\gamma}{2+\gamma} x_0 \;\; for\; n = 2 \right)$. In every case, a system of symmetric linear equations has to be solved which generically leads to a unique symmetric solution.

For $\gamma > 0$, the goods are substitutes and competition is in strategic substitutes. For $\gamma < 0$, they are complements and competition is in strategic complements. Therefore, we always have over-investment according to the naïve benchmark. (Corollary to Proposition 1 and Lemma 1). In the case of strategic substitutes we get over-investment according to the open loop benchmark in the case $\gamma < 0$ and when relation (C1+) applies (Proposition 2). (C1+) is, for n = 2, equal to

$$s > \frac{1}{2+\gamma}. \tag{29}$$

For $\gamma \geq 1$ and, for $\gamma < 0$ when relation (29) applies with “smaller instead of “larger”, over-investment occurs according to the welfare benchmark; for $\gamma < 0$ and when relation (29) applies, there is under-investment (Proposition 3). The region $0 < \gamma < 1$, is not covered by Proposition 3. A direct computation reveals $\gamma = 2/3$ to be the separating line.

Figure 4 indicates parameter regions with over- and under-investment for the three benchmarks. Some limits on γ and s are also indicated. The Hessian of the profit functions in game EC is a negative definite matrix only if $s > 1/2$. For game CC, eq.

(8) provides us with dG_i / dc_i and we find that $d^2G_i / dc_i^2 < 0$ is equivalent to $s > \frac{\gamma b(n-1)}{2}(1+(n-1)b\gamma)$, i.e. for n=2, $s > \frac{2\gamma^2}{\gamma^2 - 4}$. The second order conditions for a welfare maximum are always fulfilled. According to relation (12), stability is equivalent to $-2/(n-1) < \gamma < 2/(n-1)$. Note that Cournot competition with homogenous goods $(\gamma = 1)$ and constant marginal costs is not stable[6] for $n \geq 3$.

Note that, depending on s and γ, r must take appropriate values to guarantee interior solutions with $0 \leq c_0 \leq 1$ (see eq. (27)). So, in Figure 4, we cannot assume constant r, but for every s and γ the indications “over-/under-investment” apply for those r which guarantee interior solutions.

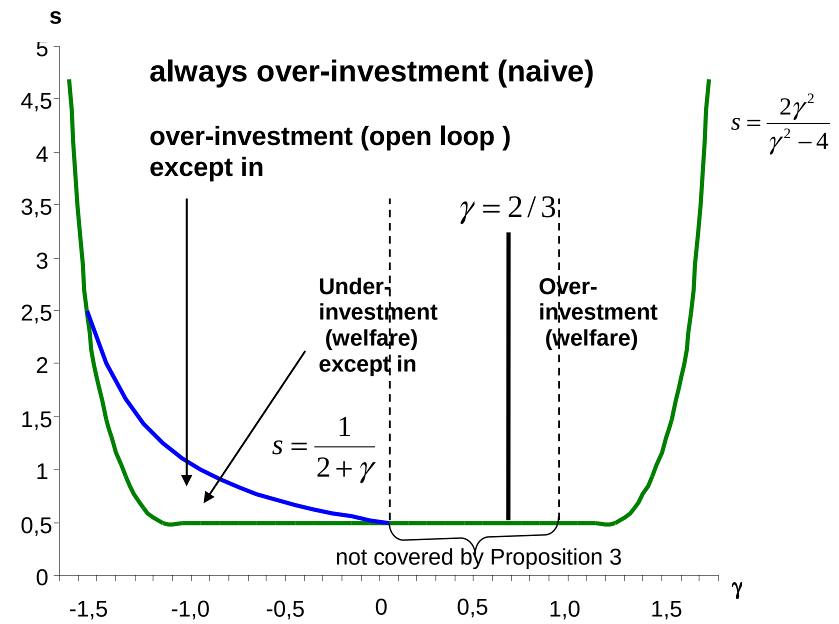

Figure 4: Quantity competition with n = 2, linear demand functions eq. (23), and constant marginal costs eq. (24). γ = 1 is the case of homogeneous goods.

[6] This problem has been addressed by Palander (1939) and Theocaris (1959).

Would we get rid of certain distinctions if we would introduce additional stability requirements? First, such requirements are far less convincing for the game EC or the first stage of the game CC. Investment is more or less irreversible while production can easily be changed. Second, they would not help. In the example, we find stability in both cases. If competition is in strategic complements, then game EC is always stable. After the redefinition of the technology variable c_i as $-c_i$, it is easy to see that EC is a supermodular game and, due to our assumption of a unique equilibrium, this equilibrium is globally stable (Vives, 2000, p. 54).

VI. Summary and conclusion

The investigation of price competition is relegated to the appendix. The joint results are summarized in Table 1 below.

The results with respect to price competition are nearly the same as for quantity competition. Note, however, that the weak assumption (A6) is not needed in the case of price competition. The conditions (C1), (C2), (C3) are rather similar but not identical. (C1-) describes cases in which all producers investing the same additional amount has a negative net effect on marginal costs (taking into account increased equilibrium production in the second stage). (C2+) and (C3+) both require goods to be sufficiently homogeneous, (C2-) and (C3-) require them to be sufficiently heterogeneous.

We can see in Table 1 that the Folk Theorem applies only for the naïve benchmark. Additional conditions determine the outcome for the other two benchmarks. It is interesting that under the welfare benchmark with sufficiently homogeneous substitutes and strategic substitutes, n = 2 may be special. Such a result is also found by Elberfeld (2003). It should be easy to generalize the investigation with respect to cooperation of the producers in the first stage. Other benchmarks may turn out to be more difficult to handle.

Naïve benchmark	**Strategic substitutes**	**Strategic complements**
Substitutes	over-investment	under-investment
Complements	under-investment	over-investment

Open Loop benchmark	**Strategic substitutes**	**Strategic complements** net inv. eff. **-**	net inv. eff. +
Substitutes	over	under	over
Complements	under	over	under

Welfare benchmark		**Strategic substitutes**	**Strategic complements** net inv. eff. -	net inv. eff. +
Substitutes	low heterogeneity	over for n > 2	under	over
	medium heterogeneity	?	?	?
	high heterogeneity	under	over	under
Complements		under for n > 2	over	under

Table 1: Under- and over-investment under different benchmarks. Net inv. eff. – or net inv. eff. + mean: (C1-) or (C1+) apply. Low heterogeneity means: (C2+) or (C3+) apply, high heterogeneity means: (C2-) or (C3-) apply.

The paper relies on assumptions (which should always apply) and conditions (which may or may not apply). Conditions (C2) and (C3) have an easily interpretable meaning. Condition (C1) is more technical. The question remains whether it can also be given a more elementary interpretation. The most restrictive assumption in this paper is symmetry. There is little hope, however, to generally deal with asymmetric cases except in the special case n = 2 or under special assumptions such as where only two technologies are available (Elberfeld, 2003). We can assume that the above results also hold under “nearly symmetric” circumstances and thus offer us a reference for principal policy decisions. For essentially asymmetric cases, only numerical simulations seem to be possible.

The message of this paper is that investment in two stage models (with a symmetry assumption) can indeed be qualitatively characterized. It is, however, necessary to explicitly specify the benchmark and, depending on the benchmark, to observe additional conditions.

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Appendix: Price competition

Demand for the (potentially) heterogeneous goods is described by $x_i(p)$, $p=(p_1,...p_n)$, $\partial x_i/\partial p_i<0$; costs are as in (1). The firms compete with prices; otherwise the games CC and EC are as described in Section II. Thus i enjoys profits $G_i=p_i x_i(p)-C_i(c_i,x_i)$. In the second stage of the CC game the (interior) best reply p_i of firm i to p_{-i} of the other firms fulfils

$$(30)\qquad \frac{\partial G_i}{\partial p_i}=x_i+\left(p_i-\frac{\partial C_i}{\partial x_i}\right)\frac{\partial x_i}{\partial p_i}=0.$$

Therefore, $x_i>0$ requires $p_i-\frac{\partial C_i}{\partial x_i}>0$. This system of best replies determines the second-stage equilibrium $p_i(c), i=1,...,n$. The first stage equilibrium $\tilde{c}$ of the game "competition with cost functions" fulfils (in the case of interior equilibria)

$$(31)\qquad \frac{dG_i}{dc_i}=\frac{\partial G_i}{\partial c_i}+\frac{\partial G_i}{\partial p_i}\cdot\frac{dp_i}{dc_i}+\sum_{k\neq i}\frac{\partial G_i}{\partial p_k}\cdot\frac{dp_k}{dc_i}$$
$$=-\frac{\partial C_i}{\partial c_i}+\left(p_i-\frac{\partial C_i}{\partial c_i}\right)\sum_{k\neq i}\frac{\partial x_i}{\partial p_k}\cdot\frac{dp_k}{dc_i}=0.$$

Contrary to (31), cost efficiency requires

$$(32)\qquad \frac{\partial G_i}{\partial c_i}=-\frac{\partial C_i}{\partial c_i}=0.$$

For equilibrium values, the difference between eqs. (31) and (32) is

$$(33)\qquad \tilde{z}_i^p=\left(p_i-\frac{\partial C_i}{\partial c_i}\right)\sum_{k\neq i}\frac{\partial x_i}{\partial p_k}\cdot\frac{\partial p_k}{\partial c_i}.$$

Because of $dx_i/dp_i<0$ and the relations (1), $\partial^2 G_i(c_i,x_i(p))/\partial p_i^2<0$, the curves $p_i(c_i)$ resulting from eqs. (31) and (32) (where $\tilde{z}_i^p=\left(p_i-\frac{\partial C_i}{\partial c_i}\right)\sum_{k\neq i}\frac{\partial x_i}{\partial p_k}\cdot\frac{\partial p_k}{\partial c_i}$ takes the equilibrium value) have the slope $\frac{dp_i}{dc_i}=-\frac{\partial^2 C_i(c_i,x_i)/\partial c_i^2}{\partial^2 C_i(c_i,x_i)/\partial x_i\partial c_i * \partial x_i/\partial p_i}>0$. $\partial C_i(c_i,x_i(p)/\partial c_i=\tilde{z}_i$ lies below (above) $\partial C_i/\partial c_i=0$ if the sign of $\tilde{z}_i^p$ is positive (negative). "Below" means that the same c_i values are connected with smaller p_i values.

Definition: Evaluated at $(\tilde{x},\tilde{c})$, we set $g:=+1$ if all $\partial x_i/\partial p_k<0$, i.e. if goods are complements and $g:=-1$ if all $\partial x_i/\partial p_k>0$, i.e. if goods are substitutes. We define $h:=-1$ *if all* $dp_i/dc_k>0$ and $h:=+1$ *if all* $dp_i/dc_k<0$.

Note that the definition of h corresponds to that in the case of quantity competition. $h:=+1$ means that increasing c_i induce decreasing prices and increasing quantities. For price competition, $\tilde{z}_i^p$ is positive for $g*h=+1$ and vice versa.

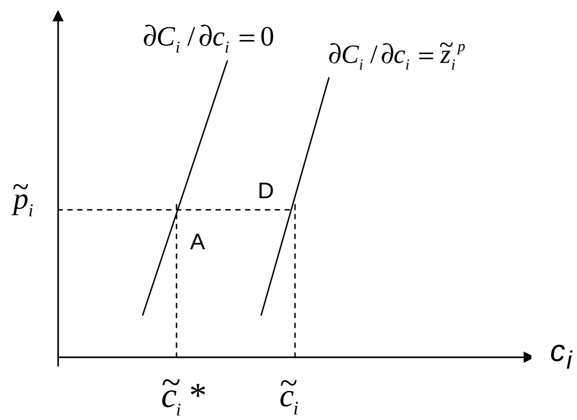

Figure 5: The equilibrium A of Game CC and the cost efficient production D in the case $\tilde{z}_i^p>0$ (case $g*h=+1$).

Proposition 4: If $g * h = +1\,(-1)$ then, according to the naïve benchmark, $\tilde{x}$ is produced with under-investment (over-investment).
Proof: See Figure 5.

As in the case of quantity competition we investigate symmetric equilibria. The derivatives dp_i / dc_k can again be computed with the Implicit Function Theorem. α and β are defined by the second derivatives of G_i with respect to prices, but otherwise the arguments are exactly the same as in Section III.

Lemma 3: The following derivatives are valuated at symmetric and stable equilibria. Then we get:

(i) $$\frac{dp_i(c_1,\ldots,c_n)}{dc_i} = a * \frac{\partial^2 C}{\partial c_i \partial x_i} * \frac{\partial x_i}{\partial p_i} > \frac{1}{\alpha} * \frac{\partial^2 C}{\partial c_i \partial x_i} * \frac{\partial x_i}{\partial p_i} > 0$$

(ii) $$\frac{dp_i(c_1,\ldots,c_n)}{dc_k} = b * \frac{\partial^2 C}{\partial c_i \partial x_i} * \frac{\partial x_i}{\partial p_i} < 0\ (>0)$$

for strategic substitutes (complements),

(iii)

$$\frac{dp_0(c_0,\ldots,c_0)}{dc_0} = (a + (n-1)b) * \frac{\partial^2 C}{\partial c_0 \partial x_0} * \frac{\partial x_i}{\partial p_i}(p_i = p_0) < (>)$$
$$\frac{1}{\alpha} * \frac{\partial^2 C}{\partial c_0 \partial x_0} * \frac{\partial x_i}{\partial p_i}(p_i = p_0)$$

for strategic substitutes (complements).

(iv) $$a + (n-1)b = \frac{\beta - \alpha}{(n-1)\beta^2 - \alpha^2 - (n-2)\alpha\beta} < \begin{Bmatrix} nb & \text{for } b<0 \\ -(n-2)b & \text{for } b>0 \end{Bmatrix} \le 0 .$$

Proof: As proof of Lemma 1. Note that in eq. (6) the second derivatives of G_i (including α and β) are defined now with respect to prices. ■

Corollary: The Folk Theorem described in the Introduction applies for the naïve benchmark.
Proof: Proposition 4 and Lemma 3 (ii).

Lemma 4: $\frac{dp_0(c_0,\ldots,c_0)}{dc_0}$, the slope of the function which describes the second stage equilibrium quantities with identical $c_i = c_0$, is larger than the slope of $\partial C_0 / \partial c_0 = const$ if

(C1p+) $$\frac{\partial C_0^2}{\partial c_0^2} > \frac{\alpha - \beta}{(n-1)\beta^2 - \alpha^2 - (n-2)\alpha\beta} \left(\frac{\partial^2 C_0}{\partial c_0 \partial x_0} \right)^2 \left(\frac{\partial x_i}{\partial p_i}(p_i = p_0) \right)^2 .$$

Proof:
Taking into account Lemma 3 (iv), relation (C1p+) is equivalent to

(34)
$$\frac{dp_0}{dc_0} = (a + (n-1)b) \frac{\partial C_0}{\partial c_0 \partial x_0} \frac{\partial x_i}{\partial p_i}(p_i = p_0) > -\frac{\partial^2 C_0 / \partial c_0^2}{\partial^2 C_0 / \partial c_0 \partial p_0 * \partial x_i / \partial p_i} = \text{slope of } \frac{\partial C_0}{\partial c_0} = 0 .$$
■

Proposition 5:
(i) *Strategic substitutes*: There is over-investment (under-investment) in Game CC according to the open loop benchmark if goods are substitutes (complements).
(ii) *Strategic complements*: If (C1p+) applies and if goods are substitutes (complements) then there is under- (over-) investment according to the open loop benchmark. If relation (C1p-) applies, we find the opposite result.

Proof: *(i):* The arguments are the same as in Proposition 2. In the benchmark game **EC** the best replies of firms fulfil eq. (30) as well as (32), i.e. firms produce with minimal costs. The second order condition for best replies is a negative definite Hessian of the profit function which implies $\partial^2 G_i / \partial c_i{}^2 * \partial^2 G_i / \partial p_i{}^2 - (\partial^2 G_i / \partial c_i \partial p_i)^2 > 0$ and which is equivalent to

(35) $$\frac{\partial^2 C_i(c_i, x_i) / \partial x_i \partial c_i * \partial x_i / \partial p_i}{\partial^2 G_i(c_i, p_i) / \partial p_i{}^2} < -\frac{\partial^2 C_i(c_i, x_i) / \partial c_i{}^2}{\partial^2 C_i(c_i, x_i) / \partial x_i \partial c_i * \partial x_i / \partial p_i} .$$

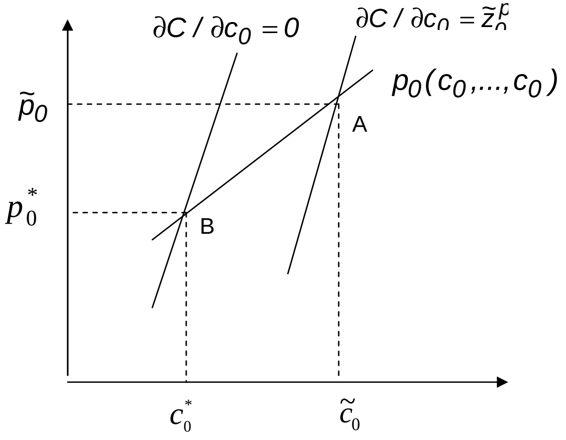

Figure 6: The equilibrium A of Game CC and the equilibrium B of Game EC in the case of complements and strategic substitutes, i.e. $\tilde{z}_i^p > 0$ ($g * h = +1$).

(ii): If (C1p+) is fulfilled then we can argue as under (i). When the relation of slopes changes, we get opposite results ■

For strategic substitutes, relation (35) and Lemma 3 (iii) show that relation (34) applies. The uniqueness of (x^*, c^*) means that $\tilde{c} < c^* (> c^*)$ applies for $\tilde{z} < (>) 0$, i.e. for substitutes (complements).

Welfare is measured as the sum of consumers' and producers' surplus, i.e.

(36) $$W = \sum_i \left[\int_{p_i(c_1,\dots,c_n)}^{\infty} x_i(\xi_i, p_{-i}(c))d\xi_i + p_i(c)x_i(p(c)) - C_i(x_i(p(c)), c_i) \right].$$

Let us assume regulatory measures only with respect to investment; the second stage of the game is still an oligopoly where prices $p_i(c)$ are chosen. An interior (second best) optimum then requires

(37) $$\frac{\partial W}{\partial c_k} = \sum_i \left[\int_{p_i(c_1,\dots,c_n)}^{\infty} \sum_{j \neq i} \frac{\partial x_i}{\partial p_j}\frac{dp_j}{dc_k} d\xi_i + \left(p_i - \frac{\partial C_i}{\partial x_i}\right)\left(\frac{\partial x_i}{\partial p_i}\frac{dp_i}{dc_k} + \sum_{j \neq i}\frac{\partial x_i}{\partial p_j}\frac{dp_j}{dc_k}\right)\right] - \frac{\partial C_k}{\partial c_k} = 0$$

for k = 1, …, n. Because of the second stage best replies eq. (2), we can substitute

(38) $$p_i - \frac{\partial C_i}{\partial x_i} = -\frac{x_i}{\partial x_i / \partial p_i}.$$

As in the case of quantity competition, we assume that the system of equations (37) has a unique and symmetric solution $\hat{c}^p = (\hat{c}_0^p, \dots, \hat{c}_0^p)$.

We say that there is under- (over-) investment with respect to the welfare benchmark if $\tilde{c}_0^p > (<) \hat{c}_0^p$. The equilibrium quantities implied by $\hat{c}^p$ are $\hat{p}^p = p(\hat{c}^p)$. $p(\hat{c}^p)$ is the same function as in the previous sections. We can now proceed as in the last section, only eqs. (31) are substituted by eqs. (37). Lemma 3 shows that $\frac{dx_j}{dc_k} = b \cdot \frac{\partial^2 C_i}{\partial x_i \partial c_i}\frac{\partial x_i}{\partial p_i}$, $\frac{dx_k}{dc_k} = a \cdot \frac{\partial^2 C_k}{\partial x_k \partial c_k}\frac{\partial x_i}{\partial p_i}$ with a, b defined in eqs. (13) and (14). For identical c_i and taking into account (38) and Lemma 2, we get ($k \neq i$):

(39) $$\hat{z}_i^p = ((n-1)b + a)\left[(n-1)\left(\frac{\partial x_i}{\partial p_i}\int_{p_i}^{\infty}\frac{\partial x_i(\xi_i, p_{-i})}{\partial p_k}d\xi_i - x_i\frac{\partial x_i}{\partial p_k}\right) - x_i\frac{\partial x_i}{\partial p_i}\right]\frac{\partial^2 C}{\partial c_i \partial x_i}.$$

For substitutes, the result of the comparison depends again on the question of how differentiated the goods are. In the case of substitutes and strategic substitutes, sufficient alternative conditions are:

(C2p+) $$(n-1)\left[\frac{\partial x_i}{\partial p_i}\int_{p_i}^{\infty}\frac{\partial x_i(\xi_i, p_{-i})}{\partial p_k}d\xi_i - x_i\frac{\partial x_i}{\partial p_k}\right] - x_i\frac{\partial x_i}{\partial p_i} \le 0$$

or

(C2p-) $$(n-1)\left[\frac{\partial x_i}{\partial p_i}\int_{p_i}^{\infty}\frac{\partial x_i(\xi_i, p_{-i})}{\partial p_k}d\xi_i - x_i\frac{\partial x_i}{\partial p_k}\right] - x_i\frac{\partial x_i}{\partial p_i} \ge \frac{n-1}{n-2}x_i\frac{\partial p_i}{\partial x_k} \quad for\ n > 2.$$

(C2p-) limits the homogeneity of the goods while (A7p+) requires sufficient homogeneity.

In case of substitutes and strategic complements, sufficient alternative conditions are:

(C3p-) $$(n-1)\left[\frac{\partial x_i}{\partial p_i}\int_{p_i}^{\infty}\frac{\partial x_i(\xi_i, p_{-i})}{\partial p_k}d\xi_i - x_i\frac{\partial x_i}{\partial p_k}\right] - x_i\frac{\partial x_i}{\partial p_i} \geq 0,$$

(C3p+)

$$(n-1)\left[\frac{\partial x_i}{\partial p_i}\int_{p_i}^{\infty}\frac{\partial x_i(\xi_i, p_{-i})}{\partial p_k}d\xi_i - x_i\frac{\partial x_i}{\partial p_k}\right] - x_i\frac{\partial x_i}{\partial p_i} \leq -\frac{n-1}{n}x_i\frac{\partial p_i}{\partial x_k} \quad .$$

Proposition 6: Let us assume that (A1) to (A5) apply.

(i) *Substitutes and strategic substitutes (for all $x_0(c^s)$):* If (C2p+) applies then there is over-investment in the CC game with respect to the welfare benchmark. If (C2p-) applies then there is under-investment.

(ii) *Substitutes and strategic complements (for all $x_0(c^s)$):* If (C3p+) and relation (C1+) apply then there is under-investment with respect to the welfare benchmark. If (C3p-) and relation (C1+) apply then there is over-investment. If relation (C1p-) applies, we get the opposite results.

(iii) *Complements and strategic substitutes (for all $x_0(c^s)$)*: There is under-investment with respect to the welfare benchmark.

(iv) *Complements and strategic complements (for all $x_0(c^s)$)*: If relation (C1p+) applies there is over-investment with respect to the welfare benchmark. If relation (C1p-) applies we get the opposite result.

Proof: See Proposition 3 and take into account that, for identical c_i, $\tilde{z}_i^p = -(n-1)b * x_i * \frac{\partial x_i}{\partial p_i} * \frac{\partial^2 C}{\partial c_i \partial x_i}$. In the case of complements, the pendent to (A6), namely $(n-1)\left(\frac{\partial x_i}{\partial p_i}\int_{p_i}^{\infty}\frac{\partial x_i(\xi_i, p_{-i})}{\partial p_k}d\xi_i - x_i\frac{\partial x_i}{\partial p_k}\right) - x_i\frac{\partial x_i}{\partial p_i} \geq -\frac{n-1}{n}x_i\frac{\partial p_i}{\partial x_k}$, is always fulfilled.